

حمل الآن

مجاناً وحصرياً

المراجعة رقم (1)

اختبار شهر اكتوبر





Quiz

1

on lesson 1 – unit 1

Total mark
10

Answer the following questions :

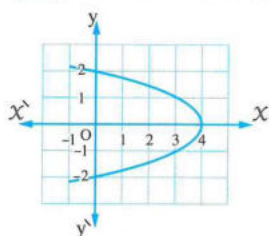
First question

4 marks

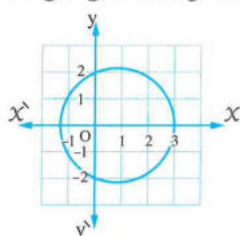
1 mark for each item

Choose the correct answer from those given :

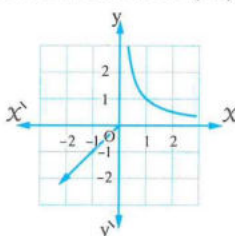
(1) Which of the following figures represents a function in (X) ?



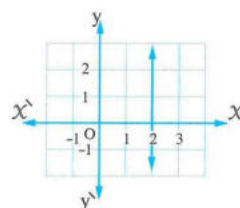
(a)



(b)



(c)



(d)

(2) The opposite figure represents a function in X whose domain is

(a) $\mathbb{R}$

(b) $\mathbb{R} -]-2, 2[$

(c) $\mathbb{R} - [-2, 2]$

(d) $\mathbb{R} - \{0\}$

(3) The opposite figure represents a function in X whose range is

(a) $\mathbb{R} - [0, 2]$

(b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - [0, 2[$

(d) $\mathbb{R} -]0, 2]$

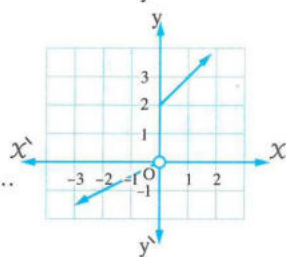
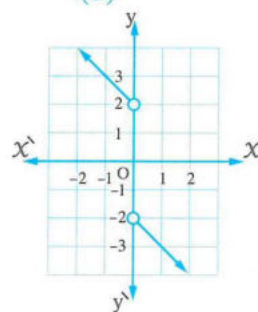
(4) If $f(X) = \sqrt{4 - x^2}$, then the domain of the function $f =$

(a) $[-2, 2]$

(b) $] -2, 2[$

(c) $[-2, 2[$

(d) $] -2, 2]$

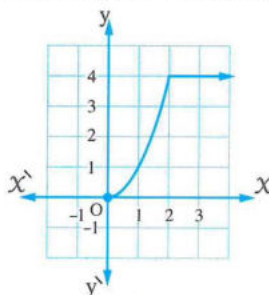


Second question

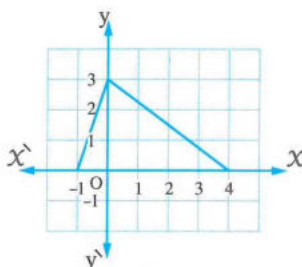
6 marks

2 marks for each item

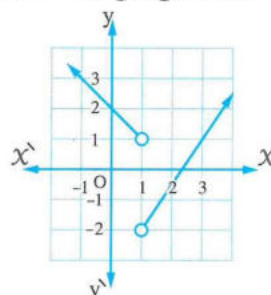
Investigate the monotony of the functions represented by the following figures :



(1)



(2)



(3)

Answer the following questions :

First question

6 marks

1 mark for each item

Choose the correct answer from those given :

(1) If $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x}$, then $(f \cdot g)(x) = \dots\dots\dots$

(a) $\sqrt{x} + \frac{1}{x}$

(b) $\sqrt{x}$

(c) $\frac{1}{\sqrt{x}}$

(d) $\sqrt{x} - \frac{1}{x}$

(2) If $f(x) = x + 1$, $g(x) = x^2$, then $(f \circ g)(2) = \dots\dots\dots$

(a) 3

(b) 4

(c) 5

(d) 9

(3) The domain of the function $f : f(x) = \sqrt{5-x}$ equals $\dots\dots\dots$

(a) $\mathbb{R} - \{5\}$

(b) $\mathbb{R}^+$

(c) $]-\infty, 5]$

(d) $[5, \infty[$

(4) If $f(x) = \sqrt{x}$, $g(x) = x^2$, then the domain of $(f \div g) = \dots\dots\dots$

(a) $[0, \infty[$

(b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R}^+$

(d) $\mathbb{R}^-$

(5) If $f(x) = \sqrt{x-1}$, $g(x) = \sqrt{1-x}$, then the domain of $(f+g)$ is $\dots\dots\dots$

(a) $[1, \infty[$

(b) $]-\infty, 1]$

(c) $[-1, \infty[$

(d) $\{1\}$

(6) If the relation between x , $f(x)$, $g(x)$ is as

shown in the given table for some values of

x , then the value of x that satisfies that

$g(f(x)) = -1$ is $\dots\dots\dots$

(a) 8

(b) 3

(c) 2

(d) 4

x	$f(x)$	$g(x)$
-1	-2	4
zero	0	3
1	2	2
2	4	1
3	6	0
4	8	-1

Second question

4 marks

2 marks for each item

If $f(x) = \frac{1}{x}$, $g(x) = x + 3$, find :

(1) $(f \circ g)(x)$

(2) $(g \circ f)(x)$

Answer the following questions :

First question

6 marks

1 mark for each item

Choose the correct answer from those given :

(1) If f is an even function in the interval $[a, b]$, then $b = \dots\dots\dots$

- (a) a (b) $-a$ (c) $2a$ (d) a^3

(2) The domain of the function $f : f(x) = \frac{5}{\sqrt[3]{x-8}}$ is $\dots\dots\dots$

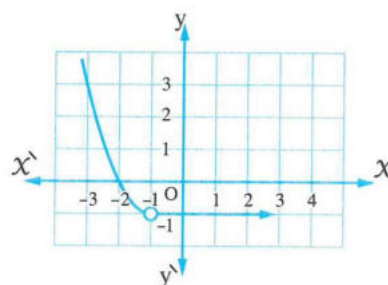
- (a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{8\}$ (d) $[8, \infty[$

(3) The odd function from the given ones is $f : f(x) = \dots\dots\dots$

- (a) $x \sin x$ (b) $\cos x$ (c) 5 (d) $x \cos x$

(4) The range of the function represented in the opposite figure is $\dots\dots\dots$

- (a) $\mathbb{R} - \{-1\}$
(b) $[-1, \infty[$
(c) $] -1, \infty[$
(d) $\mathbb{R}$



(5) If f is a one - to - one function and the point $(2, 3)$ belongs to the function f , which of the following points could belong to the function f ?

- (a) $(5, 3)$ (b) $(2, -1)$ (c) $(3, 2)$ (d) All the previous.

(6) If $f(x) = x^3$, $g(x) = x^2 + 1$, which of the following is an odd function ?

- (I) $(f \times g)$ (II) $(f \circ g)$ (III) $(g \circ f)$
(a) (I) only. (b) (II) and (III) (c) (I) and (II) (d) (I) and (III)

Second question

4 marks

If $f_1(x) = x^5$, $f_2(x) = \sin x$, find $(f_1 + f_2)(x)$ hence

find the type of $(f_1 + f_2)$ whether it is even, odd or otherwise.

Answer the following questions :

First question 3 marks

Graph the function $f : f(x) = \begin{cases} |x| & , \quad x \leq 0 \\ x^2 & , \quad x > 0 \end{cases}$ from the graph , deduce the range of the function and determine its type whether it is even , odd or otherwise and investigate its monotony.

Second question 2 marks

Find the domain of the function $f : f(x) = \frac{2x+1}{x-2}$ and prove that f is one - to - one.

Third question 3 marks

If $f(x) = x^2 - 1$, $g(x) = x + 1$, graph the function $\frac{f}{g}$, determine the domain and the range of the function then investigate its monotony.

Fourth question 2 marks

Graph the function $f : f(x) = \begin{cases} x-1 & , \quad 2 < x \leq 4 \\ -1 & , \quad -2 \leq x \leq 2 \end{cases}$ from the graph , deduce the range and investigate the monotony.

Answer the following questions :

First question

6 marks

1 mark for each item

Choose the correct answer from those given :

- (1) The curve of the function $f : f(x) = x^2 + 4$ is the same curve of the function $g : g(x) = x^2$ by translation of a magnitude 4 units in the direction of
- (a) $\overrightarrow{OX}$ (b) $\overrightarrow{OY}$ (c) $\overrightarrow{Ox}$ (d) $\overrightarrow{Oy}$
- (2) The function which is one - to - one from the functions defined by the following rules is
- (a) $f_1(x) = x + 2$ (b) $f_2(x) = x^2$
 (c) $f_3(x) = |x|$ (d) $f_4(x) = 5$
- (3) If f is a function where $f(x) = \frac{1}{x}$, then the point of symmetry of the function $f(x+1)$ is
- (a) (1 , 0) (b) (0 , 1) (c) (-1 , 0) (d) (-1 , 1)
- (4) If $f(x) = \sqrt{x+4}$, $g(x) = x^2 - 4$, then $(f \circ g)(x) = \dots\dots\dots$
- (a) $|x|$ (b) x^2 (c) $x^2 + 4$ (d) 2
- (5) The domain of a real function $f(x)$ is $[-2, 3]$, then the domain of the function $g(x) = f(x-2)$ is
- (a) $[-2, 3]$ (b) $[-4, 1]$ (c) $[0, 5]$ (d) $\mathbb{R}$
- (6) If $f(x)$ is an odd function, then $|f(x)|$ is
- (a) odd. (b) even.
 (c) both odd and even. (d) neither odd nor even.

Second question

4 marks

Graph the function $f : f(x) = |4 - x^2|$ from the graph, deduce the range of the function and determine its type whether it is even, odd or otherwise and investigate its monotony.



Quiz 1 on lesson 1 – unit 3

Total mark
10

Answer the following questions :

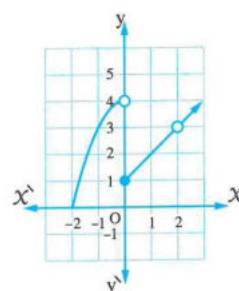
First question

4 marks

1 mark for each item

From the opposite figure , find :

- (1) $f(\text{zero}^+)$ (2) $f(\text{zero}^-)$
(3) $f(2)$ (4) $\lim_{x \rightarrow 2} f(x)$



Second question

6 marks

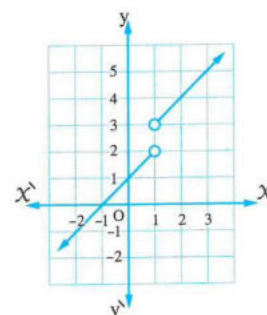
2 marks for each item

Choose the correct answer from those given :

- (1) The opposite figure represents the graph of the function f , then

$$\lim_{x \rightarrow 1} f(x) = \dots\dots\dots$$

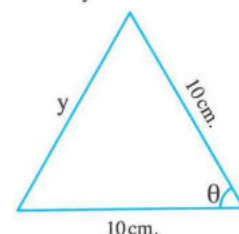
- (a) 2 (b) 3
(c) 1 (d) not exist.



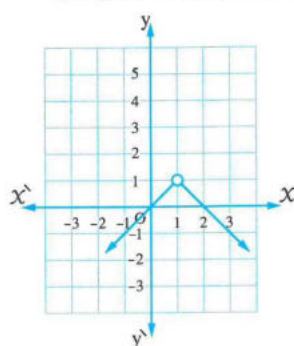
- (2) In the opposite figure :

At $\theta \rightarrow \frac{\pi}{2}$, then $y \rightarrow \dots\dots\dots$ cm.

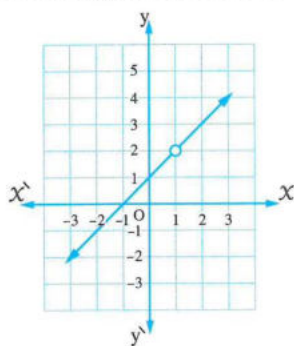
- (a) zero (b) 5
(c) 10 (d) $10\sqrt{2}$



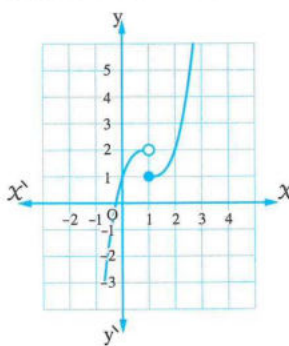
- (3) Which of the following functions has no limit at $x = 1$?



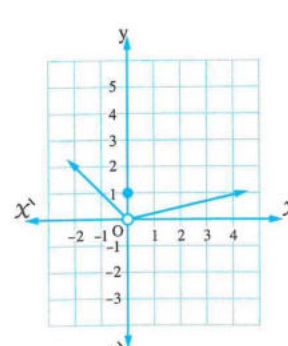
(a)



(b)



(c)



(d)

Answer the following questions :

First question

2 marks

$\frac{1}{2}$ mark for each item

Choose the correct answer from those given :

(1) $\lim_{x \rightarrow 0} \frac{1+x}{4x-1} = \dots\dots\dots$

(a) -1

(b) $\frac{1}{4}$

(c) $-\frac{1}{4}$

(d) 1

(2) $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3} = \dots\dots\dots$

(a) -6

(b) zero

(c) 3

(d) 6

(3) The opposite figure represents $f(x)$

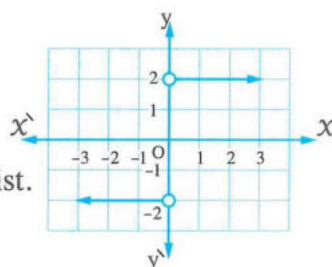
, then $\lim_{x \rightarrow 2} f(x) = \dots\dots\dots$

(a) 0

(b) -2

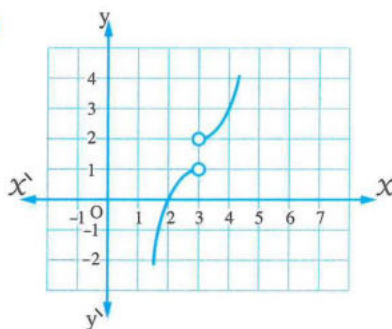
(c) 2

(d) not exist.

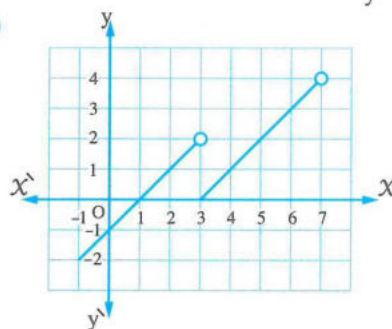


(4) Which of the following functions has a limit at $x=3$?

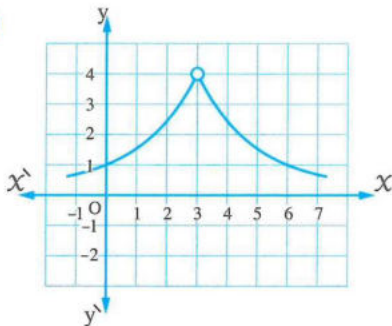
(a)



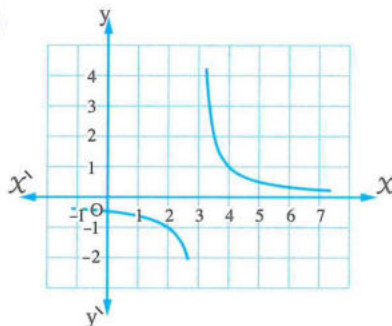
(b)



(c)



(d)



Second question

8 marks

2 marks for each item

Find :

(1) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{x^2+x}$

(2) $\lim_{x \rightarrow 1} \frac{3x^2+x-4}{x^2-1}$

(3) $\lim_{x \rightarrow 4} \frac{x^3-15x-4}{x-4}$

(4) $\lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{3}{x^3-1} \right)$



Quiz

1

on lesson 1 – unit 4

 Total mark
10

Answer the following questions :

First question

6 marks

1½ mark for each item

Choose the correct answer from those given :

(1) In $\triangle ABC$, if $a = 6$ cm. , $m(\angle B) = 2 m(\angle A) = 80^\circ$, then $c = \dots\dots\dots$ cm.

(a) $\frac{6 \sin 40^\circ}{\sin 60^\circ}$

(b) $\frac{\sin 60^\circ}{6 \sin 40^\circ}$

(c) $\frac{\sin 40^\circ}{6 \sin 60^\circ}$

(d) $\frac{6 \sin 60^\circ}{\sin 40^\circ}$

(2) ABC is an equilateral triangle, its side length equals $8\sqrt{3}$ cm. , then the length of the diameter of its circumcircle equals $\dots\dots\dots$ cm.

(a) 8

(b) $16\sqrt{3}$

(c) 16

(d) $4\sqrt{3}$

(3) In $\triangle XYZ$, if $\frac{2x}{\sin X} = 8$ cm. , then the area of its circumcircle equals $\dots\dots\dots$ cm^2

(a) 16π

(b) 8π

(c) 4π

(d) 64π

(4) In $\triangle ABC$, $\frac{\sin(A+B)}{\sin A + \sin B} = \dots\dots\dots$

(a) 1

(b) $\frac{c}{a+b}$

(c) $\frac{a}{a+c}$

(d) $\frac{b}{a+c}$

Second question

4 marks

ABC is a triangle in which : $c = 19$ cm. , $m(\angle A) = 112^\circ$, $m(\angle B) = 33^\circ$

Find to the nearest 2 decimal places each of b and the radius length of its circumcircle.



Test 1

1 Choose the correct answer from those given :

(1) $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9} = \dots\dots\dots$

(a) $\frac{1}{3}$

(b) $\frac{1}{6}$

(c) $\frac{1}{5}$

(d) $\frac{1}{4}$

(2) The domain of the function $f : f(x) = \frac{\sqrt{x-2}}{x-4}$ is $\dots\dots\dots$

(a) $[2, \infty[$

(b) $]-\infty, 2]$

(c) $[2, \infty[- \{4\}$

(d) $]-\infty, 2] - \{4\}$

(3) All the following functions is one - to - one on its domain except $f(x) = \dots\dots\dots$

(a) $3x$

(b) $\frac{1}{x}$

(c) 5

(d) x^3

(4) If the radius of the circumcircle of triangle ABC equals 3 cm.

and $\sin A + \sin B + \sin C = 2$, then the perimeter of the triangle = $\dots\dots\dots$ cm.

(a) 6

(b) 9

(c) 12

(d) 24

(5) The greatest side in the given triangle $\approx \dots\dots\dots$ cm.

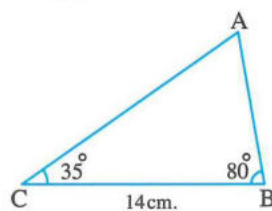
(to the nearest integer)

(a) 20

(b) 16

(c) 15

(d) 14



(6) If $\lim_{x \rightarrow 1} \frac{x^2 - k}{x + 2} = -1$, then $k = \dots\dots\dots$

(a) 4

(b) -2

(c) 2

(d) ± 2

(7) The function $f : f(x) = |x + 2|$ is decreasing on the interval $\dots\dots\dots$

(a) $]0, \infty[$

(b) $[2, \infty[$

(c) $]-2, \infty[$

(d) $]-\infty, -2[$

(8) If f is an odd function , then $\frac{7f(3) + 3f(-3)}{2f(-3)} = \dots\dots\dots$

(a) 2

(b) -2

(c) 5

(d) -0.5

(9) The solution set of the equation : $|x - 7| = 2x - 2$ in $\mathbb{R}$ is $\dots\dots\dots$

(a) $\{3, -5\}$

(b) $\{3\}$

(c) $\{5\}$

(d) $\emptyset$

(10) The symmetric point on the curve of the function $f : f(x) = \frac{x+1}{x}$ is $\dots\dots\dots$

(a) $(0, 1)$

(b) $(1, 0)$

(c) $(0, 0)$

(d) $(1, -1)$

(11) $\lim_{x \rightarrow 2} \frac{(x+1)^4 - 81}{x-2} = \dots\dots\dots$

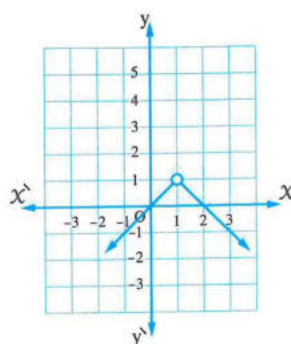
(a) 18

(b) 81

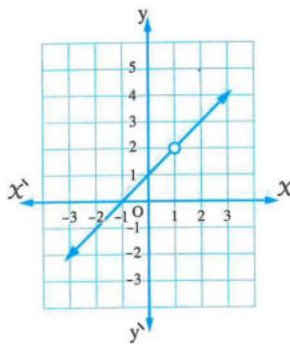
(c) -108

(d) 108

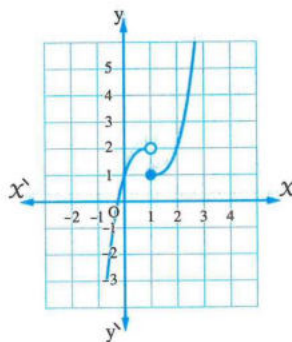
(12) Which of the following functions has no limit at $x = 1$?



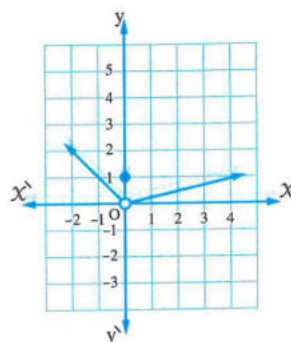
(a)



(b)



(c)



(d)

2 Answer the following two questions :

(1) If $f(x) = x^2 - 3$, $g(x) = \sqrt{x-2}$, find $(f \circ g)(x)$ in the simplest form ,
then find $(f \circ g)(3)$

(2) Find : $\lim_{x \rightarrow 1} \frac{\sqrt{4x+5} - 3}{x-1}$

1 Choose the correct answer from those given :

(1) In $\triangle ABC$, $\frac{4b}{\sin B} = \dots\dots\dots r$, where r is the radius of the circle passes through the vertices of triangle ABC .

(a) 4

(b) 8

(c) $\frac{1}{2}$

(d) $\frac{1}{8}$

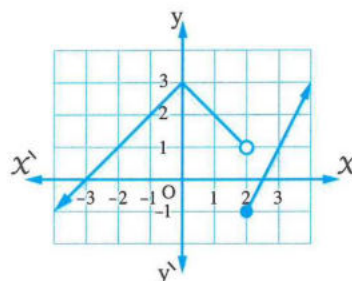
(2) The opposite figure represents the curve of the function f , then $\lim_{x \rightarrow 2} f(x) = \dots\dots\dots$

(a) 3

(b) 1

(c) -1

(d) not exist.



(3) If $f(x) = |x - 2| + 4$, then the solution set of the equation $f(x + 2) = 6$ is $\dots\dots\dots$

(a) $\{0, 4\}$

(b) $\{2, -2\}$

(c) $\{2, 4\}$

(d) $\{-2, -4\}$

(4) If f is an even function and the curve of the function passes through the point $(-3, 2m + 1)$ and $f(3) = 5$, then $m = \dots\dots\dots$

(a) zero

(b) -1

(c) 1

(d) 2

(5) The domain of the function $f : f(x) = \sqrt{x - 3}$ is $\dots\dots\dots$

(a) $\mathbb{R}$

(b) $\mathbb{R} - \{3\}$

(c) $[3, \infty[$

(d) $]-\infty, 3[$

(6) If $f(x) = \sqrt{x}$, $g(x) = x^2 - 1$, then $(f \circ g)(-3) = \dots\dots\dots$

(a) 2

(b) -4

(c) $2\sqrt{2}$

(d) undefined.

(7) $\lim_{x \rightarrow 7} \frac{x^2 - 49}{7 - x} = \dots\dots\dots$

(a) 14

(b) -49

(c) 49

(d) -14

(8) The point of symmetry of the function $f : f(x) = (x - 2)^3 + 1$ is $\dots\dots\dots$

(a) $(2, 1)$

(b) $(2, -1)$

(c) $(-2, 1)$

(d) $(-1, -2)$

(9) The opposite figure represents the function $f : f(x) = \dots\dots\dots$

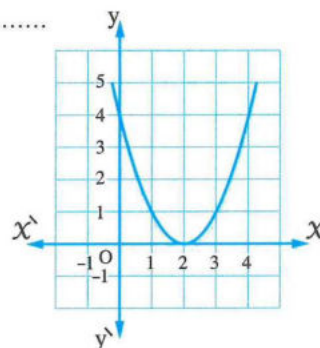
where $f : \mathbb{R} \longrightarrow \mathbb{R}$

(a) $x^2 + 2$

(b) $(x + 2)^2$

(c) $x^2 - 4x + 4$

(d) $-(x - 2)^2$



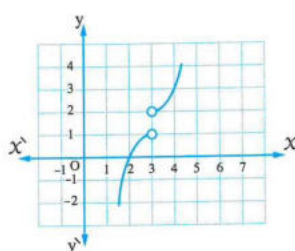
(10) If $\lim_{x \rightarrow 3} \frac{x^2 - l}{x - 3} = m$, then $(l, m) = \dots\dots\dots$

- (a) $(3, 3)$ (b) $(-9, 0)$ (c) $(9, 6)$ (d) $(0, 0)$

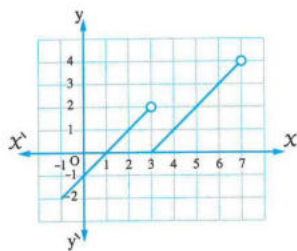
(11) In ΔXYZ , if $m(\angle X) : m(\angle Y) : m(\angle Z) = 2 : 3 : 1$, then $x : y : z = \dots\dots\dots$

- (a) $3 : \sqrt{2} : 1$ (b) $\sqrt{3} : 2 : 1$ (c) $\sqrt{2} : \sqrt{3} : 1$ (d) $2 : 3 : 1$

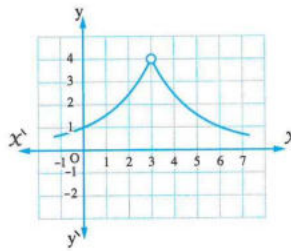
(12) Which of the following functions has a limit at $x = 3$?



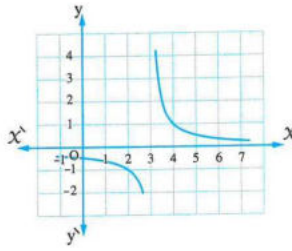
(a)



(b)



(c)



(d)

2 Answer the following two questions :

(1) Graph the curve of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = |x| + 1$. From the graph find the range and discuss the monotony and determine its type whether it is odd, even or neither.

(2) Find : $\lim_{x \rightarrow 2} \left(\frac{x^3 - 8}{x^5 - 32} + \frac{x^4 - 16}{x^7 - 128} \right)$

Test

1

Total mark

20

1 Choose the correct answer from those given :

(12 marks)

1 $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9} = \dots\dots\dots$

- (a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{1}{4}$

2 The domain of the function $f : f(x) = \frac{\sqrt{x-2}}{x-4}$ is $\dots\dots\dots$

- (a) $[2, \infty[$ (b) $] -\infty, 2]$ (c) $[2, \infty[- \{4\}$ (d) $] -\infty, 2] - \{4\}$

3 All the following functions is one - to - one on its domain except $f(x) = \dots\dots\dots$

- (a) $3x$ (b) $\frac{1}{x}$ (c) 5 (d) x^3

4 If the radius of the circumcircle of triangle ABC equals 3 cm.

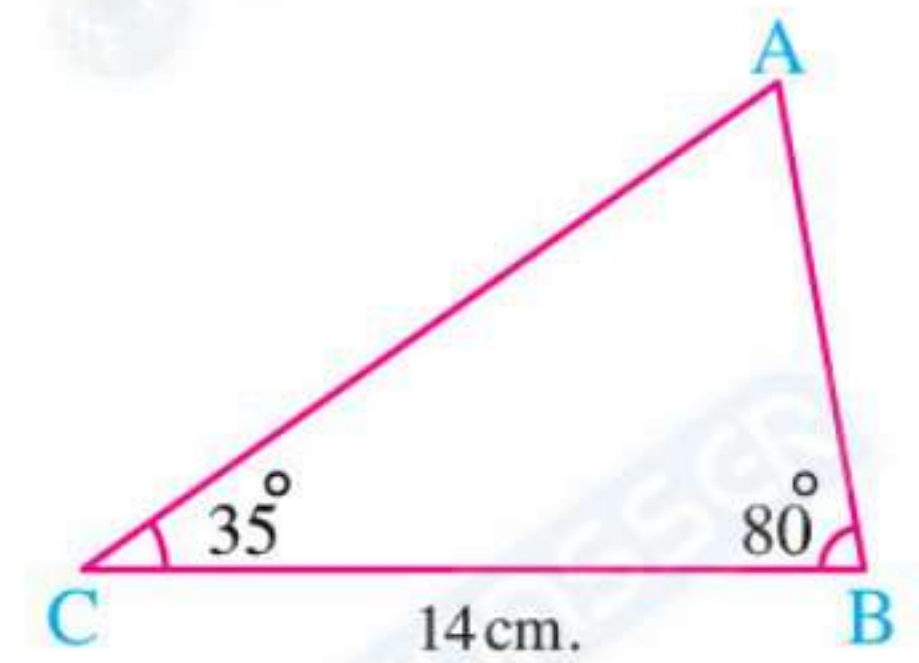
and $\sin A + \sin B + \sin C = 2$, then the perimeter of the triangle = $\dots\dots\dots$ cm.

- (a) 6 (b) 9 (c) 12 (d) 24

5 The greatest side in the given triangle $\approx \dots\dots\dots$ cm.

(to the nearest integer)

- (a) 20 (b) 16
(c) 15 (d) 14



6 If $\lim_{x \rightarrow 1} \frac{x^2 - k}{x + 2} = -1$, then $k = \dots\dots\dots$

- (a) 4 (b) -2 (c) 2 (d) ± 2

7 The function $f : f(x) = |x + 2|$ is decreasing on the interval $\dots\dots\dots$

- (a) $]0, \infty[$ (b) $[2, \infty[$ (c) $] -2, \infty[$ (d) $] -\infty, -2[$

8 If f is an odd function , then $\frac{7f(3) + 3f(-3)}{2f(-3)} = \dots\dots\dots$

- (a) 2 (b) -2 (c) 5 (d) -0.5

9 If $f(x) = x - 3$, $g(x) = x^2$, then $(f \circ g)(5) = \dots\dots\dots$

- (a) 2 (b) 4 (c) 22 (d) 25

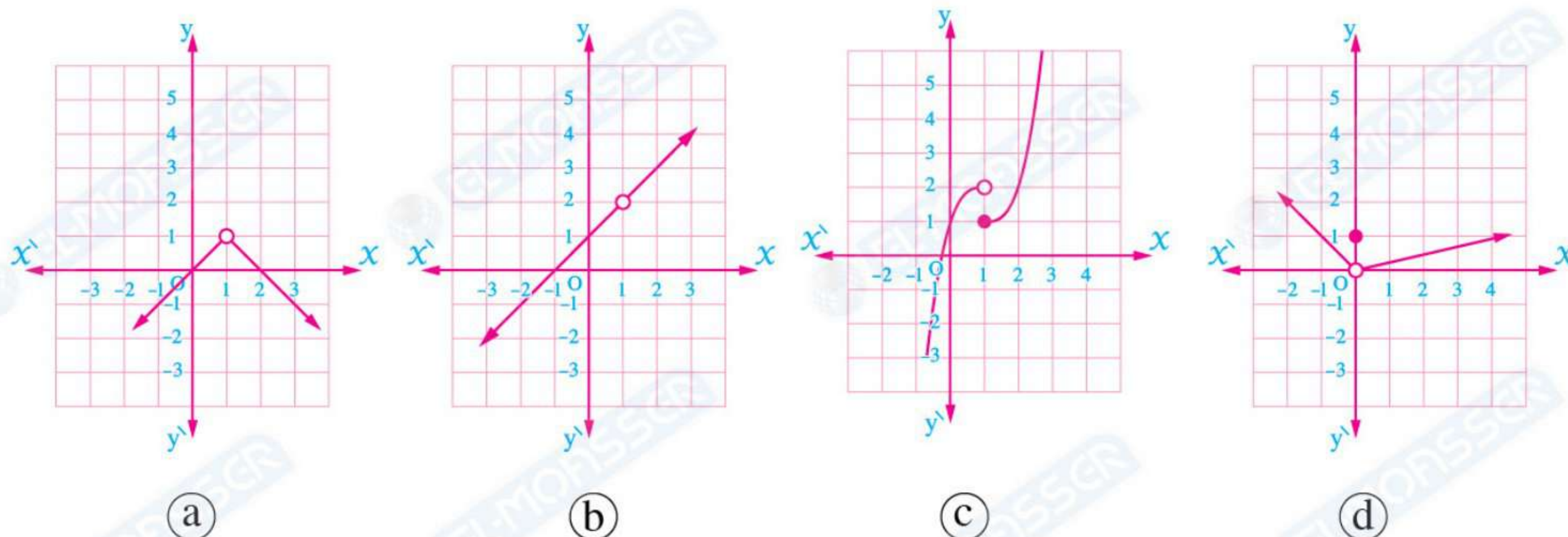
10 The symmetric point on the curve of the function $f : f(x) = \frac{x+1}{x}$ is $\dots\dots\dots$

- (a) (0 , 1) (b) (1 , 0) (c) (0 , 0) (d) (1 , -1)

11 $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} = \dots\dots\dots$

- (a) 2 (b) 4 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

12 Which of the following functions has no limit at $x = 1$?



2 Answer the following questions :

1 Graph the function $f : f(x) = \begin{cases} x^2 + 1 & , x > 0 \\ -x^2 - 1 & , x < 0 \end{cases}$

from the graph determine the range of the function and discuss its monotony.

(2 marks)

2 If $f(x) = x^2 - 3$, $g(x) = \sqrt{x-2}$, find $(f \circ g)(x)$ in the simplest form , determining its domain , then find $(f \circ g)(3)$

(2 marks)

3 Find : $\lim_{x \rightarrow 1} \frac{\sqrt{4x+5} - 3}{x-1}$

(2 marks)

4 ABCD is a parallelogram in which $m(\angle A) = 50^\circ$, $m(\angle DBC) = 70^\circ$ and $BD = 8$ cm.

Find the perimeter of the parallelogram to the nearest whole number. (2 marks)

Test

2

Total mark

20

(12 marks)

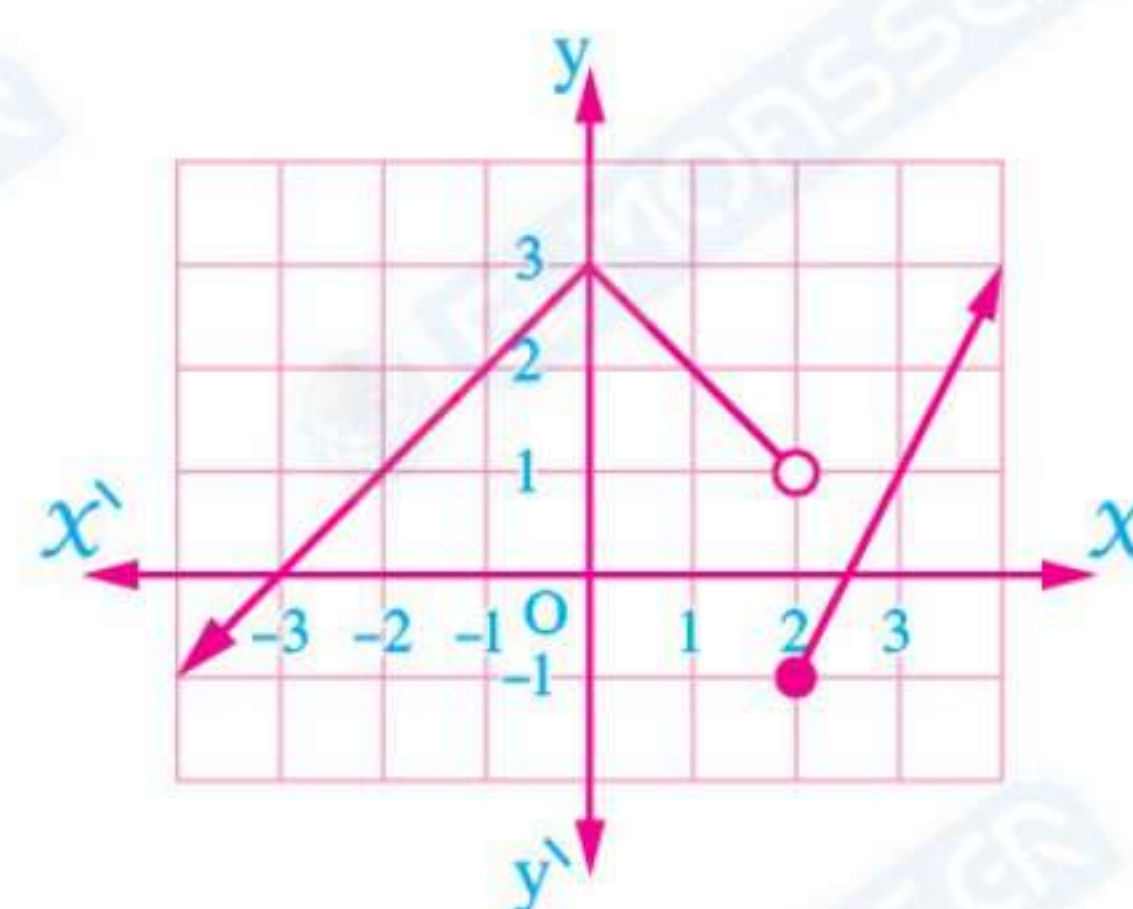
1 Choose the correct answer from those given :

- 1 In $\triangle ABC$, $\frac{4b}{\sin B} = \dots\dots\dots r$, where r is the radius of the circle passes through the vertices of triangle ABC .

(a) 4 (b) 8 (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

- 2 The opposite figure represents the curve of the function f , then $\lim_{x \rightarrow 2} f(x) = \dots\dots\dots$

(a) 3
(b) 1
(c) -1
(d) not exist.



- 3 The one - to - one function from the functions defined by the following rules is $\dots\dots\dots$

(a) $f(x) = x^2$ (b) $f(x) = -2$ (c) $f(x) = |x|$ (d) $f(x) = \frac{1}{x}$

- 4 If f is an even function and the curve of the function passes through the point $(-3, 2m + 1)$ and $f(3) = 5$, then $m = \dots\dots\dots$

(a) zero (b) -1 (c) 1 (d) 2

- 5 The domain of the function $f : f(x) = \sqrt{x - 3}$ is $\dots\dots\dots$

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $[3, \infty[$ (d) $] - \infty, 3[$

- 6 If $f(x) = \sqrt{x}$, $g(x) = x^2 - 1$, then $(f \circ g)(-3) = \dots\dots\dots$

(a) 2 (b) -4 (c) $2\sqrt{2}$ (d) undefined

- 7 $\lim_{x \rightarrow 7} \frac{x^2 - 49}{7 - x} = \dots\dots\dots$

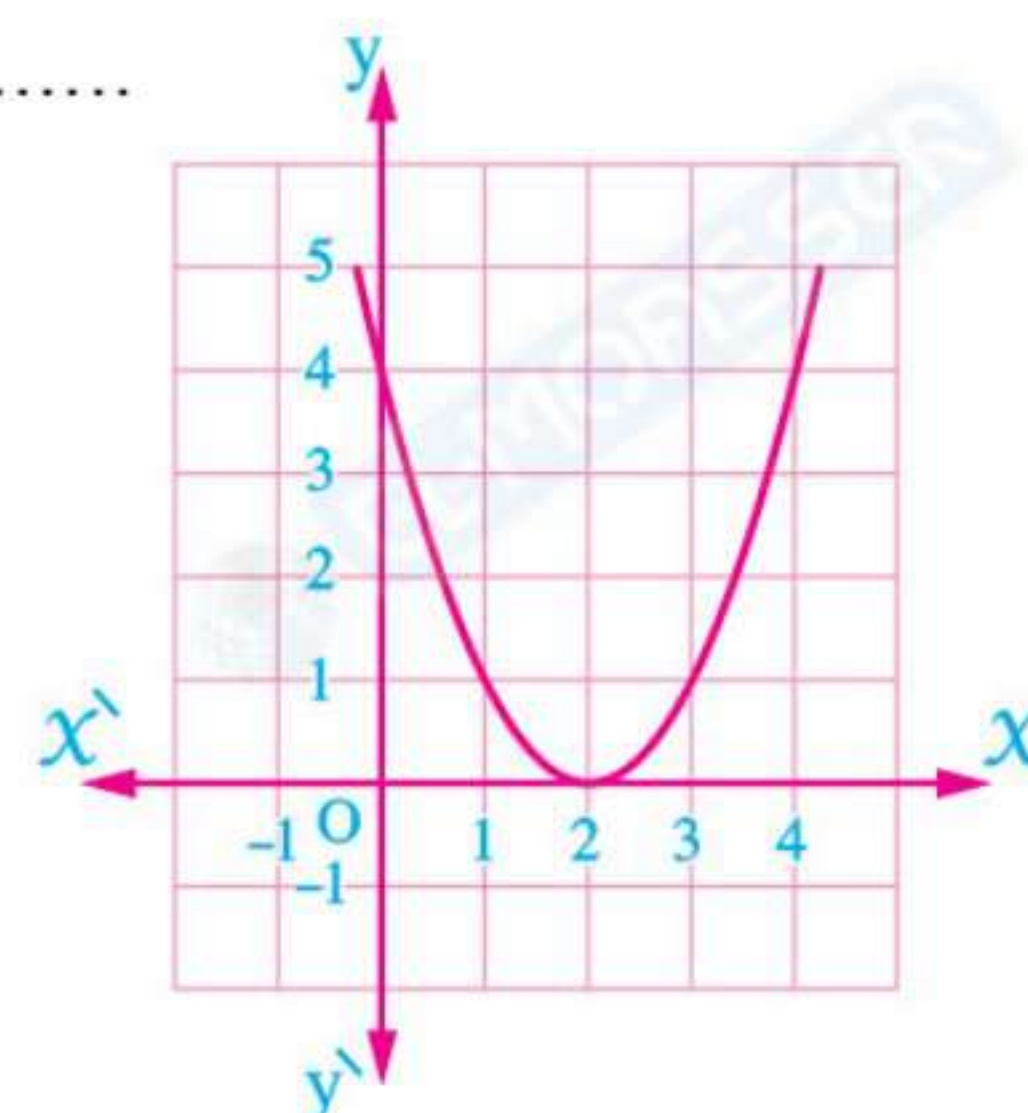
(a) 14 (b) -49 (c) 49 (d) -14

- 8 The point of symmetry of the function $f : f(x) = (x - 2)^3 + 1$ is $\dots\dots\dots$

(a) (2, 1) (b) (2, -1) (c) (-2, 1) (d) (-1, -2)

- 9 The opposite figure represents the function $f : f(x) = \dots\dots\dots$ where $f : \mathbb{R} \rightarrow \mathbb{R}$

(a) $x^2 + 2$ (b) $(x + 2)^2$
(c) $x^2 - 4x + 4$ (d) $-(x - 2)^2$



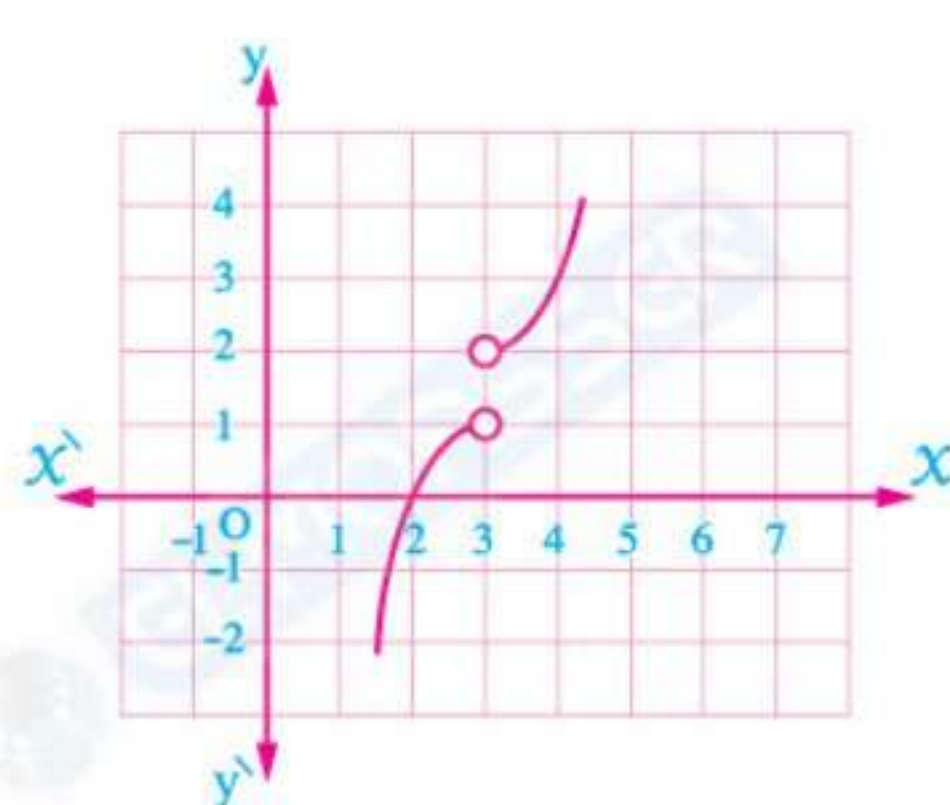
10 If $\lim_{x \rightarrow 3} \frac{x^2 - l}{x - 3} = m$, then $(l, m) = \dots\dots\dots$

- (a) (3, 3) (b) (-9, 0) (c) (9, 6) (d) (0, 0)

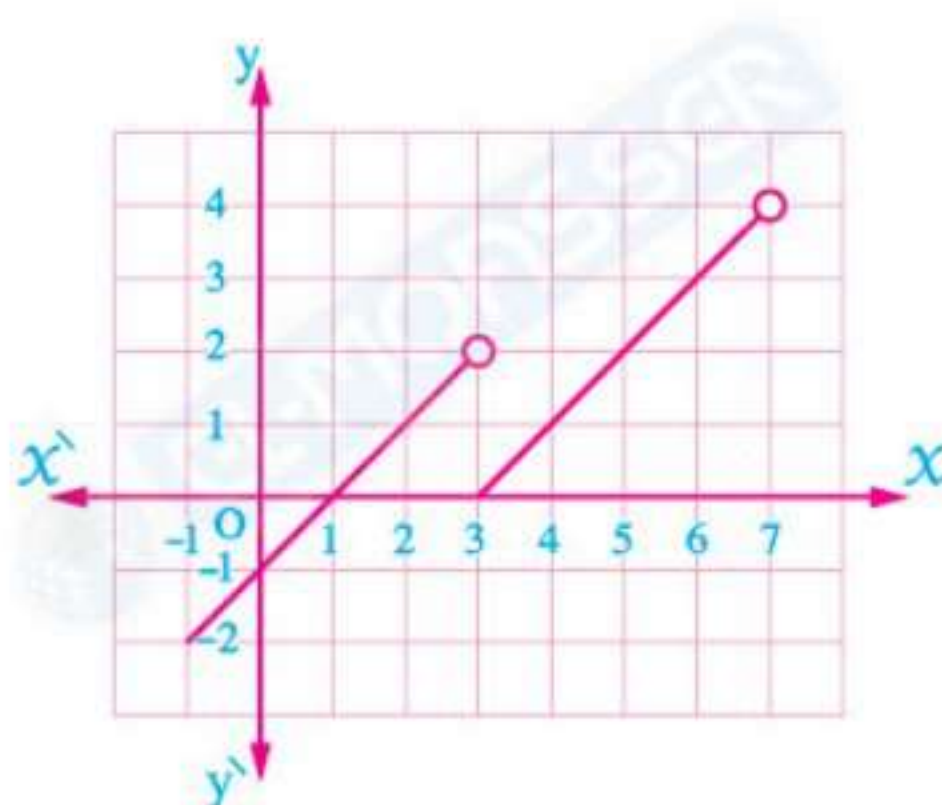
11 In ΔXYZ , if $m(\angle X) : m(\angle Y) : m(\angle Z) = 2 : 3 : 1$, then $x : y : z = \dots\dots\dots$

- (a) $3 : \sqrt{2} : 1$ (b) $\sqrt{3} : 2 : 1$ (c) $\sqrt{2} : \sqrt{3} : 1$ (d) $2 : 3 : 1$

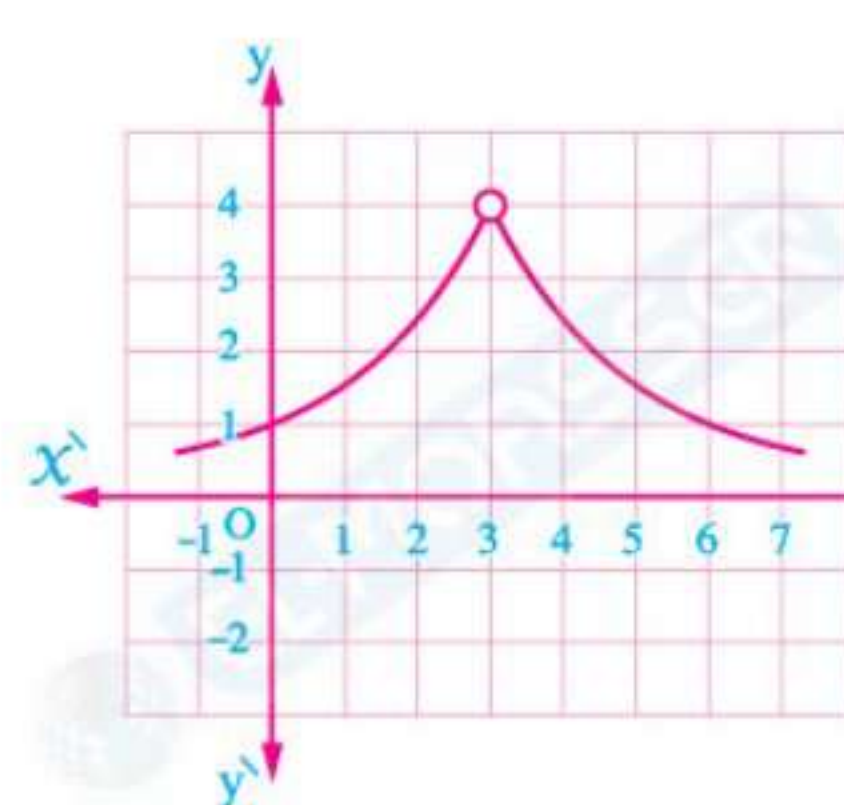
12 Which of the following functions has a limit at $x = 3$?



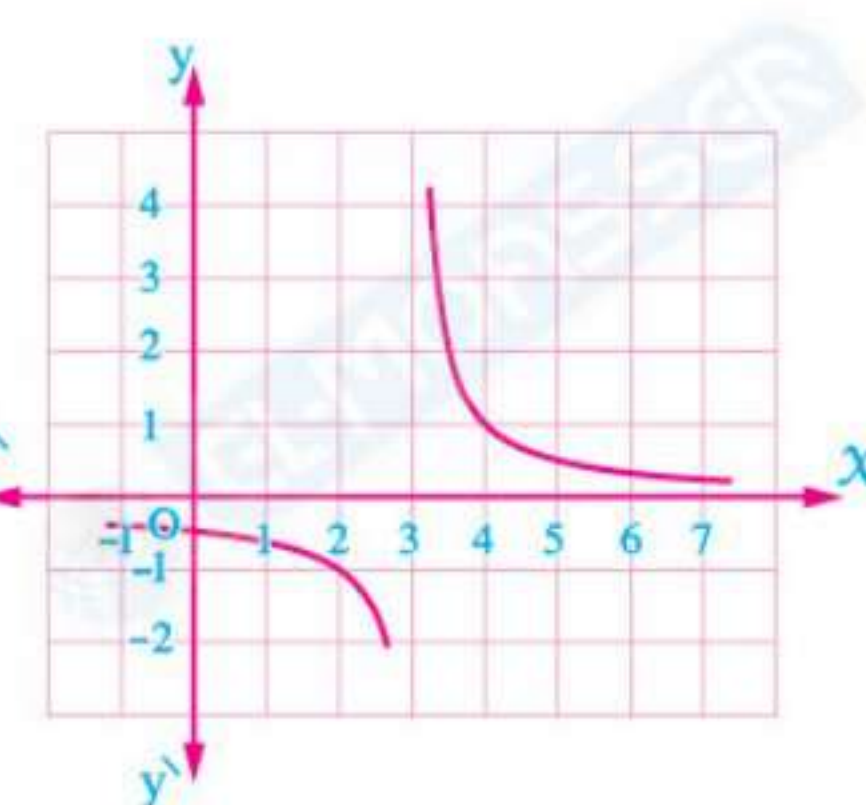
(a)



(b)



(c)



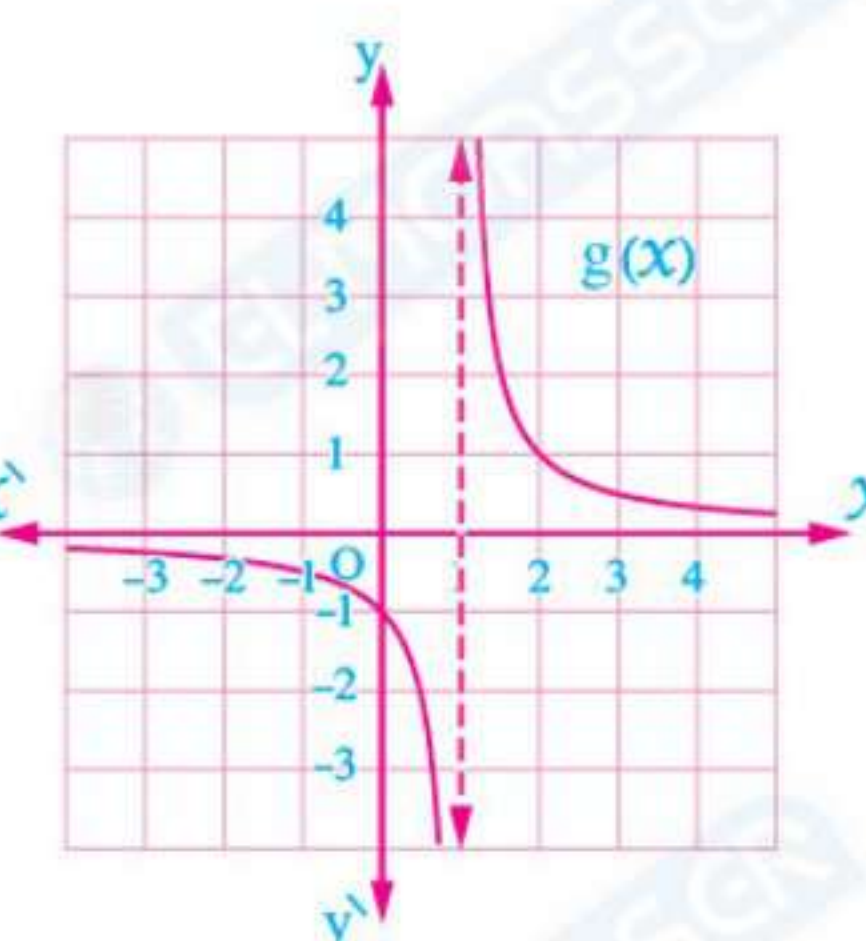
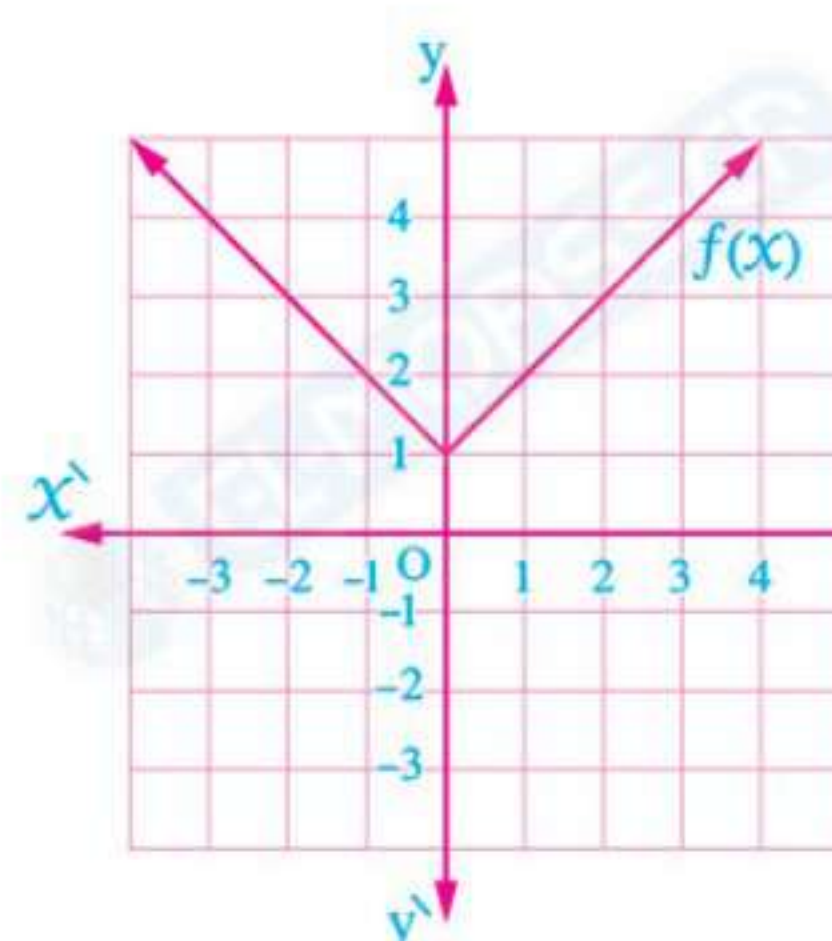
(d)

2 Answer the following questions :

1 Graph the curve of the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = |x| + 1$. From the graph find the range and discuss the monotony and determine its type whether it is odd, even or neither. (2 marks)

2 In the opposite figure :

Find $(f \circ g)(x)$ and state its domain



(2 marks)

3 Find : $\lim_{x \rightarrow 5} \frac{\sqrt{x+11} - 4}{x^2 - 25}$

(2 marks)

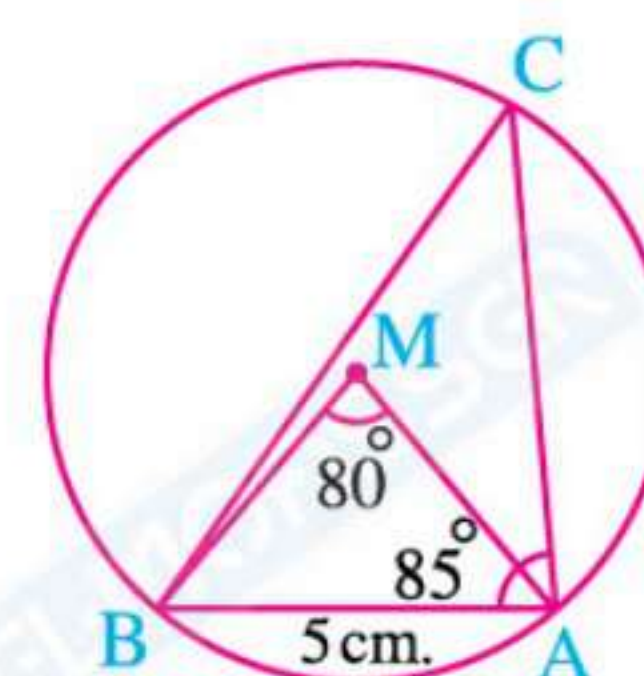
4 In the opposite figure :

A circle M, $AB = 5$ cm., $m(\angle AMB) = 80^\circ$

and $m(\angle CAB) = 85^\circ$

Find : (1) The perimeter of ΔABC .

(2) The area of the circle M.



(2 marks)

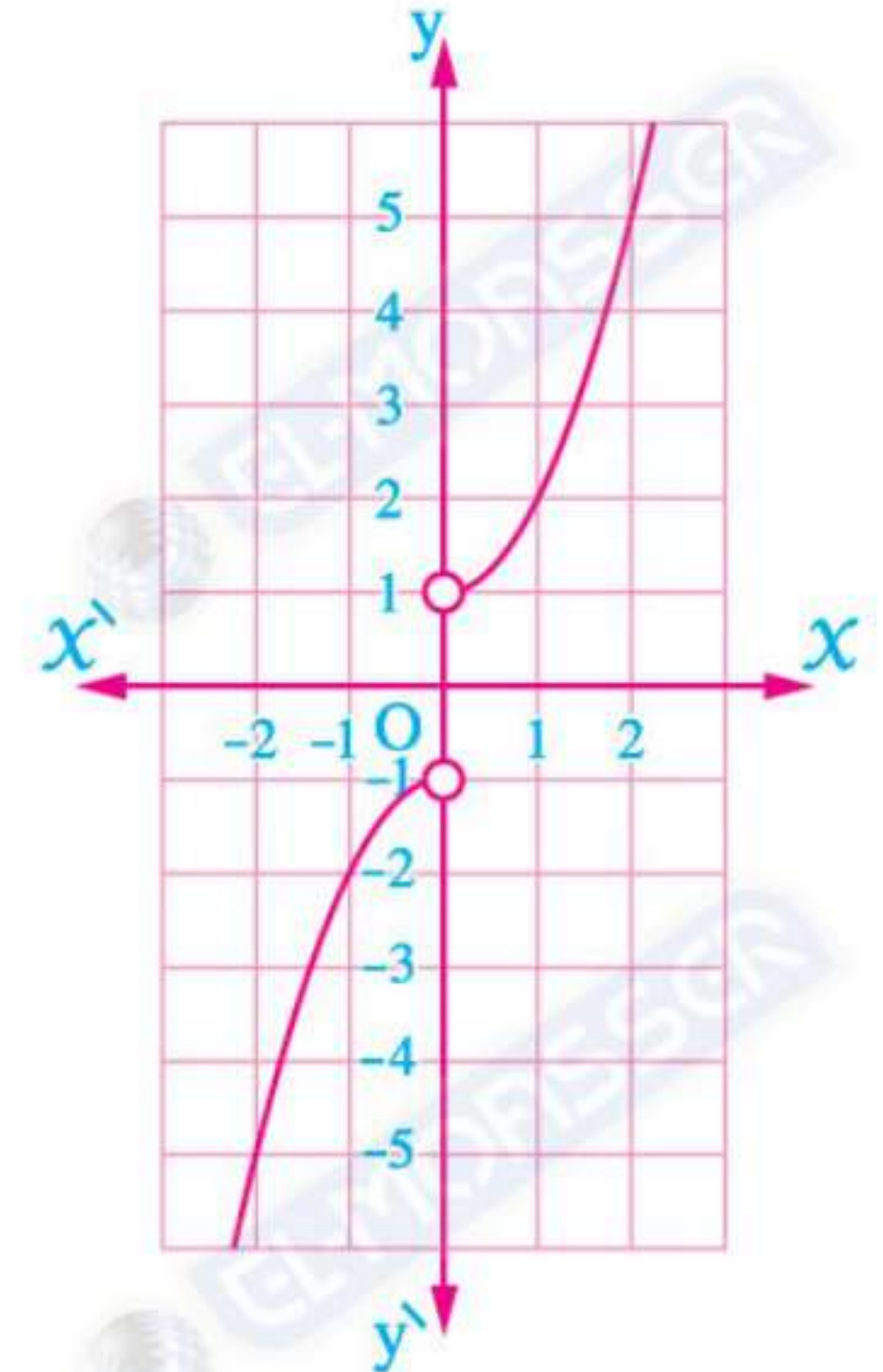
Answers of Test

1

- 1 (b) 2 (c) 3 (c) 4 (c) 5 (c) 6 (a) 7 (d) 8 (b)
9 (c) 10 (a) 11 (d) 12 (c)

2 (1) From the graph :

- The range of the function $f = \mathbb{R} - [-1, 1]$
- The function is increasing on $\mathbb{R} - \{0\}$



$$(2) \because (f \circ g)(x) = f(g(x)) = f(\sqrt{x-2}) = (\sqrt{x-2})^2 - 3 = x - 2 - 3 = x - 5$$

$$, \because D_1 = \text{domain of } g = [2, \infty[$$

, the values of x which makes $g(x)$ in the domain of f is $D_2 = \mathbb{R}$

$$\therefore \text{The domain } (f \circ g) = D_1 \cap D_2 = [2, \infty[, (f \circ g)(3) = 3 - 5 = -2$$

$$(3) \lim_{x \rightarrow 1} \frac{\sqrt{4x+5}-3}{x-1} \times \frac{\sqrt{4x+5}+3}{\sqrt{4x+5}+3} = \lim_{x \rightarrow 1} \frac{4x+5-9}{(x-1)(\sqrt{4x+5}+3)}$$

$$= \lim_{x \rightarrow 1} \frac{4(x-1)}{(x-1)(\sqrt{4x+5}+3)} = \frac{4}{6} = \frac{2}{3}$$

(4) $\because$ ABCD is a parallelogram

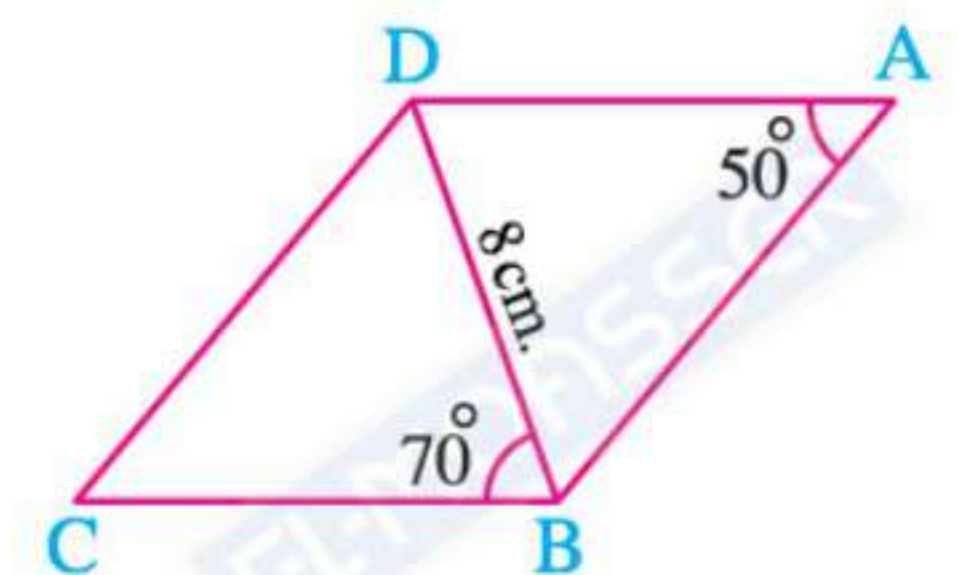
$$\therefore m(\angle C) = 50^\circ$$

$$\text{In } \triangle BDC : m(\angle BDC) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$$

$$\therefore \frac{8}{\sin 50^\circ} = \frac{BC}{\sin 60^\circ} = \frac{DC}{\sin 70^\circ}$$

$$\therefore BC \approx 9 \text{ cm.}, DC \approx 9.8 \text{ cm.}$$

$$\therefore \text{The perimeter of the parallelogram} = 2(BC + CD) \approx 38 \text{ cm.}$$

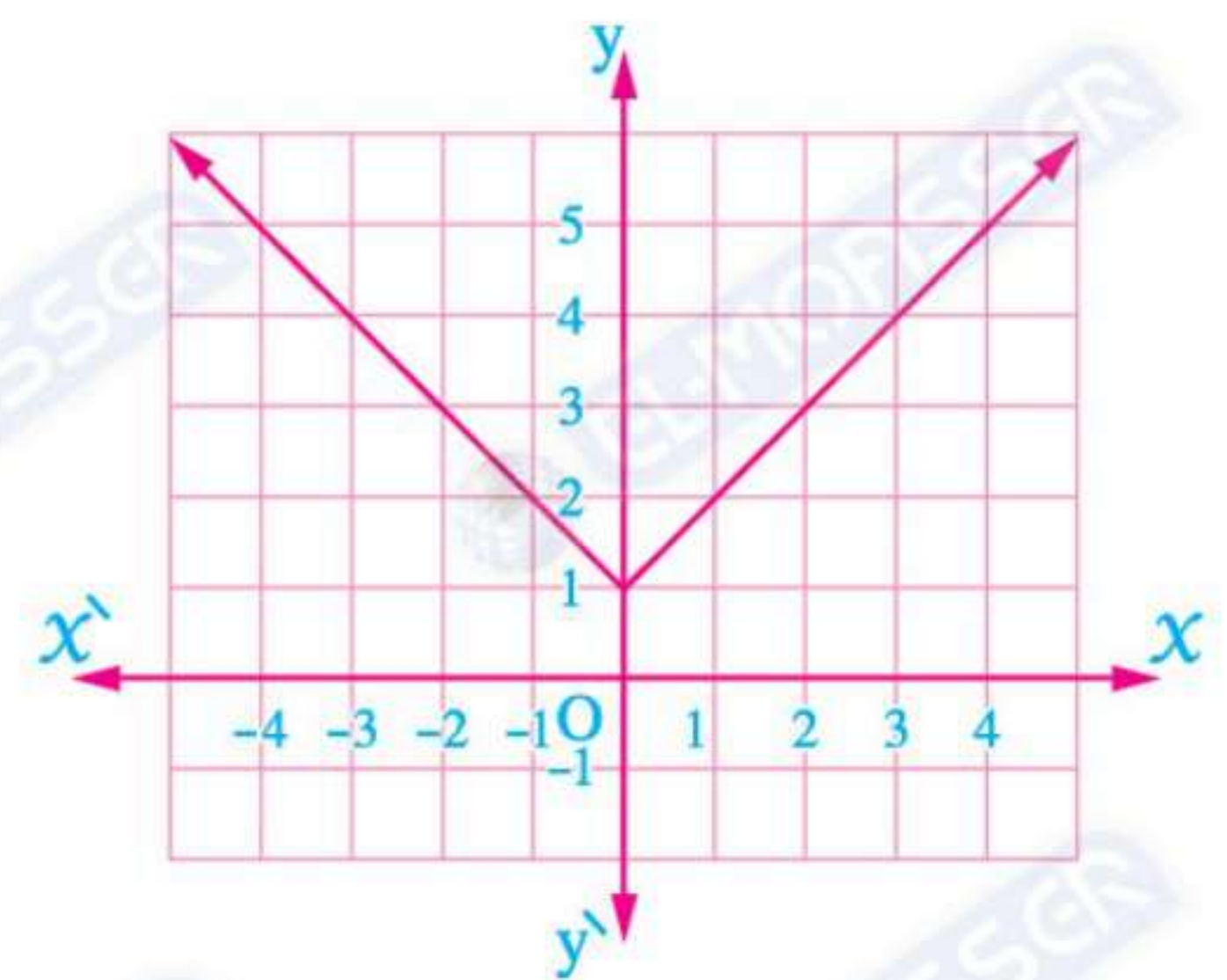


Answers of Test 2

- 1 (1) (b) 2 (d) 3 (d) 4 (d) 5 (c) 6 (c) 7 (d)
8 (a) 9 (c) 10 (c) 11 (b) 12 (c)

2 (1) From the figure :

- The range = $[1, \infty[$
- The monotony :
The function is decreasing on $]-\infty, 0[$
and increasing on $]0, \infty[$
- The type : Even.



(2) $f(x) = |x| + 1$, $g(x) = \frac{1}{x-1}$

$$\therefore (f \circ g)(x) = f[g(x)] = f\left(\frac{1}{x-1}\right) = \left|\frac{1}{x-1}\right| + 1$$

$$\therefore D_1 = \text{domain of } g = \mathbb{R} - \{1\}$$

, the values of x which makes $g(x)$ in the domain of f is $D_2 = \mathbb{R}$

$$\therefore \text{Domain of } (f \circ g) = D_1 \cap D_2 = \mathbb{R} - \{1\}$$

$$\begin{aligned} (3) \lim_{x \rightarrow 5} \frac{\sqrt{x+11}-4}{x^2-25} \times \frac{\sqrt{x+11}+4}{\sqrt{x+11}+4} &= \lim_{x \rightarrow 5} \frac{x+11-16}{(x-5)(x+5)(\sqrt{x+11}+4)} \\ &= \lim_{x \rightarrow 5} \frac{1}{(x+5)(\sqrt{x+11}+4)} = \frac{1}{80} \end{aligned}$$

(4) In $\triangle ABC$: $m(\angle C) = \frac{1}{2} m(\angle AMB) = 40^\circ$

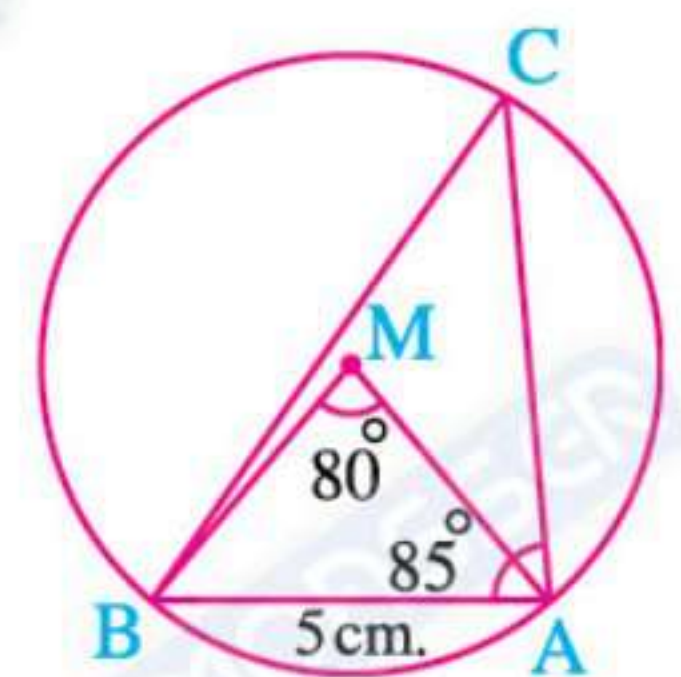
$$\therefore m(\angle ABC) = 180^\circ - (40^\circ + 85^\circ) = 55^\circ$$

$$\therefore \frac{5}{\sin 40^\circ} = \frac{b}{\sin 55^\circ} = \frac{a}{\sin 85^\circ} = 2r = \frac{\text{Perimeter of } \triangle ABC}{\sin 40^\circ + \sin 55^\circ + \sin 85^\circ}$$

$$\therefore \text{Perimeter of } \triangle ABC \approx 19.12 \text{ cm.}$$

$$, r = 3.89 \text{ cm.}$$

$$\therefore \text{The area of the circle } M = \pi r^2 = \pi (3.89)^2 \approx 47.5 \text{ cm}^2$$



كيفية طباعة صفحات معينة من ملف معين مثلا ازاي نطبع الصفحات من صفحة 4 الى صفحة 9



حمل الآن

مجاناً وحصرياً

المراجعة رقم (2)

اختبار شهر اكتوبر



Lesson:(1) Functions of real variable

Definition: The function is a relation between two variables x and y such that for every value of x there exists one and only one value of y .

For examples: $y = x^2 - 1$ is a function because:

When x say 2

$\therefore y = 5$ (one value)

The relations:

1) $y = x^2 + 5x - 3$

3) $y^3 = 5x^2 - 1$

5) $y = 5$

7) $F(x) = \begin{cases} 3x - 2 & x < 1 \\ x^2 - 1 & x \geq 1 \end{cases}$

2) $y = \sqrt{x^2 + 1}$

4) $F(x) = 1 - 2x$

6) $y = \sin x$

Are all functions

But the relations:

1) $y^2 = x^2 + 5x - 3$

3) $x = 7$

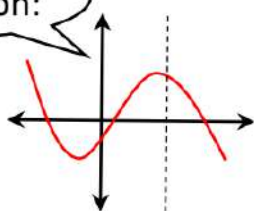
5) $y = \begin{cases} 3x - 2 & x \leq 1 \\ x^2 - 1 & x \geq 1 \end{cases}$

2) $y^4 = 2x^3 - 1$

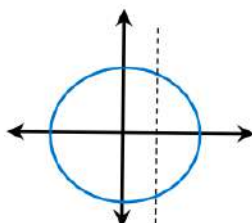
4) $y = \pm\sqrt{x+4}$

Are not functions

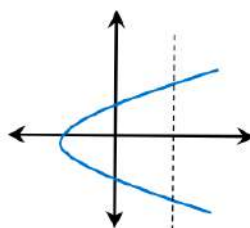
By graph:



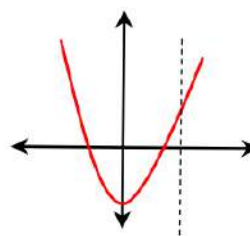
Function



Not Function



Not Function



Function

To graph the curve of any function, we must know:

1) Its domain

2) Its co-domain

3) Its rule

Given:

f : Domain $\longrightarrow$ Co-domain,

$y = f(x)$

Required:

1) graph

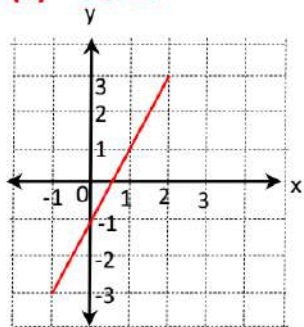
2) Domain

3) Range

Examples

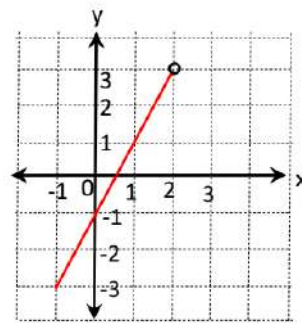
1) $F: [-1, 2] \longrightarrow \mathbb{R}, F(x) = 2x - 1$

x:	-1	0	2
F(x):	-3	-1	3

Domain = $[-1, 2]$ The range = $[-3, 3]$ 

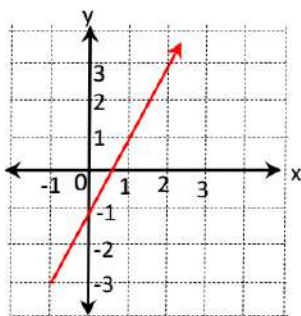
2) $F: [-1, 2[\longrightarrow \mathbb{R}, F(x) = 2x - 1$

x:	-1	0	2
F(x):	-3	-1	3

Domain = $[-1, 2[$ The range = $[-3, 3[$ 

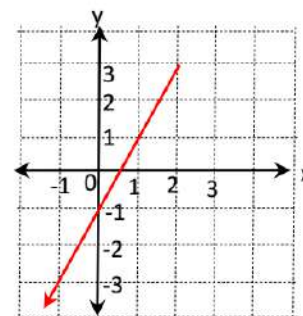
3) $F: [-1, \infty[\longrightarrow \mathbb{R}, F(x) = 2x - 1$

x:	-1	0	2
F(x):	-3	-1	3

Domain = $[-1, \infty[$ The range = $[-3, \infty[$ 

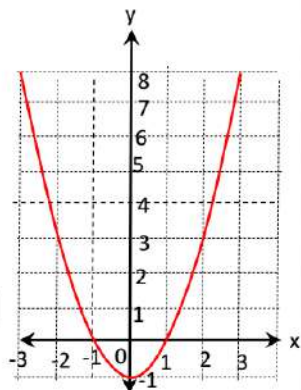
4) $F:]-\infty, 2] \longrightarrow \mathbb{R}, F(x) = 2x - 1$

x:	-1	0	2
F(x):	-3	-1	3

Domain = $]-\infty, 2]$ The range = $]-\infty, 3]$ 

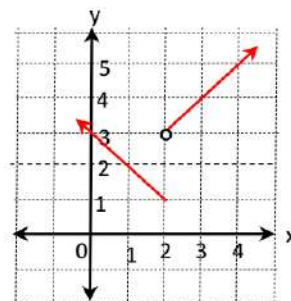
5) $F: [-3, 3] \longrightarrow [-3, 9], F(x) = 2x - 1$

x	-3	-2	-1	0	1	2	3
F(x)	8	3	0	-1	0	3	8

Domain = $[-3, 3]$ The range = $[-1, 8]$ 

6) $f(x) = \begin{cases} 3 - x, & x \leq 2 \\ x + 1, & x > 2 \end{cases}$

x	0	1	2	2	3	4
f(x)	3	2	1	3	4	5

Domain = $\mathbb{R}$ The range = $[1, \infty[$ 

The domain algebraically

[1] If $F(x)$ is a polynomial function

$$\therefore D(F) = \mathbb{R}$$

i.e. $F(x) = 13$

$$F(x) = 2x + 1$$

$$F(x) = x^2 + 3$$

$$F(x) = 2x^3 + 2x - 3$$

$$F(x) = \sqrt[3]{x^2 + 9x}$$

$$F(x) = |x + 1|$$

Domain (D) = $\mathbb{R}$

[2] If $F(x)$ is a rational function where $f(x) = \frac{h(x)}{g(x)}$, then $D(F) = \mathbb{R} - \{\text{zeroes of } g(x)\}$

Examples

1) $F(x) = \frac{3}{x}$

$$\therefore D = \mathbb{R} - \{0\}$$

2) $F(x) = \frac{x^2 - 2x}{x}$

$$\therefore D = \mathbb{R} - \{0\}$$

3) $F(x) = \frac{x^2 + 3x + 2}{x - 1}$

$$x - 1 = 0$$

$$\therefore x = 1$$

$$\therefore D = \mathbb{R} - \{1\}$$

4) $F(x) = \frac{5}{x^2 - x - 2}$

$$x^2 - x - 2 = 0$$

$$\therefore (x - 2)(x + 1) = 0 \quad \therefore x = 2, x = -1$$

$$\therefore D = \mathbb{R} - \{2, -1\}$$

5) $F(x) = \frac{x - 1}{x^2 + 4}$

$$x^2 + 4 = 0 \text{ could not be factorized}$$

$\therefore$ there are no real roots in the D_r

$$\therefore D = \mathbb{R}$$

6) $F(x) = \frac{1}{x} + \frac{x}{x^2 - 4}$

$$\therefore D = \mathbb{R} - \{0, 2, -2\}$$

[3] Domain of irrational functions " the n^{th} root "

If $f(x) = \sqrt[n]{g(x)}$ where $n \in \mathbb{Z}^+$, $g(x)$ is a polynomial

First : when (n) is an odd number, then the domain of $f = \mathbb{R}$

Second : when n is an even number, then:

If $\sqrt{\quad} \Rightarrow \begin{cases} \text{Numerator} & \therefore \text{under root} \geq 0 \\ \text{Denominator} & \therefore \text{under root} > 0 \end{cases}$

Examples

1) $F(x) = \sqrt{x-3}$

$x-3 \geq 0 \quad \therefore x \geq 3 \quad \therefore D = [3, \infty[$

2) $F(x) = \frac{2}{\sqrt{x-2}}$

$x-2 > 0 \quad \therefore x > 2 \quad \therefore D =]2, \infty[$

3) $F(x) = \frac{2}{\sqrt{4-x}}$

$4-x > 0 \quad \therefore -x > -4 \quad (\times -1)$
 $x < 4 \quad \therefore D =]-\infty, 4[$

4) 1) $F(x) = 4 - \sqrt{3-2x}$

$3-2x \geq 0 \quad \therefore -2x \geq -3 \quad \therefore x \leq \frac{3}{2}$

$\therefore D =]-\infty, \frac{3}{2}]$

5) $F(x) = \frac{2}{\sqrt{2+3x}}$

$2+3x > 0 \quad \therefore 3x > -2 \quad \therefore x > -\frac{2}{3} \quad \therefore D =]-\frac{2}{3}, \infty[$

6) $F(x) = \sqrt[3]{6-x^2}$

$\therefore$ the index of the root is odd $\therefore D = \mathbb{R}$

7) $F(x) = \sqrt{x^2-4x+4}$

$x^2-4x+4 \geq 0 \quad \therefore (x-2)^2 \geq 0 \quad \therefore D = \mathbb{R}$

8) $F(x) = \frac{1}{\sqrt{x^2-3x-4}}$

$x^2-3x-4 > 0 \quad \therefore (x-4)(x+1) > 0$

$\therefore D = \mathbb{R} -]-1, 4[$



Find the domain of the following functions

$$9) F(x) = \frac{2x-1}{x+2}$$

$$10) F(x) = \frac{x-1}{2x-1}$$

$$11) F(x) = \frac{1}{x^2-3x}$$

$$12) F(x) = \frac{x}{x^2-2x-3}$$

$$13) F(x) = \frac{x}{3x^2-x-2}$$

$$14) F(x) = \frac{x^2-1}{x^3-27}$$

$$15) F(x) = \sqrt{x}$$

$$16) F(x) = \sqrt{x-2}$$

$$17) F(x) = \sqrt{3-x}$$

$$18) F(x) = \sqrt[3]{2x+3}$$

$$19) F(x) = \frac{5}{\sqrt{x}}$$

$$20) F(x) = \frac{5}{\sqrt{x-3}}$$

$$21) F(x) = \sqrt{x^2 - 9}$$

$$22) F(x) = \sqrt{x^2 - x - 6}$$

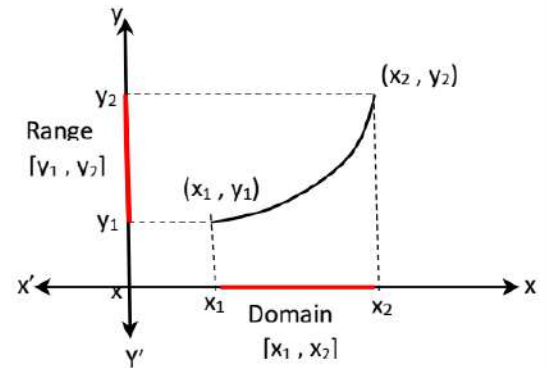
$$23) F(x) = \sqrt{4x^2 - 12x + 9}$$

$$24) F(x) = \frac{1}{\sqrt{2+x-x^2}}$$

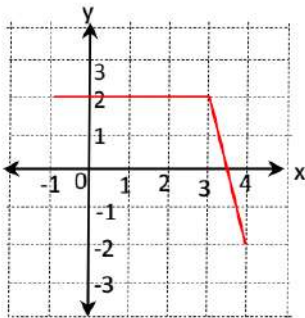
Identify the domain of the function from the graph

(1) Domain of the function is the set of the x-coordinates of all the points lie on the curve of the function

(2) Range of the function is the set of the y-coordinates of all the points lie on the curve of the function

Find the domain and the range of the following functions:

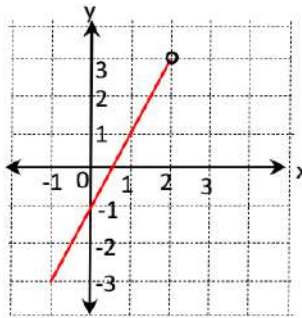
(1)



Domain =

Range =

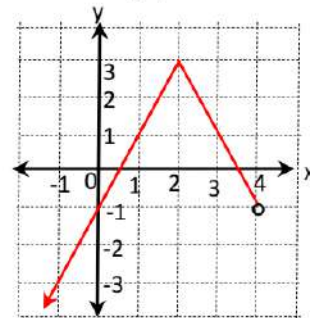
(2)



Domain =

Range =

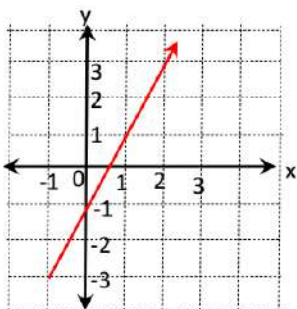
(3)



Domain =

Range =

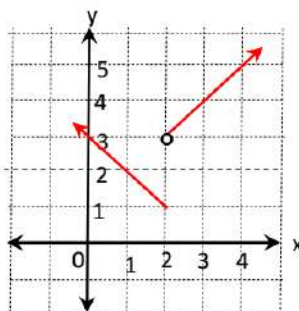
(4)



Domain =

Range =

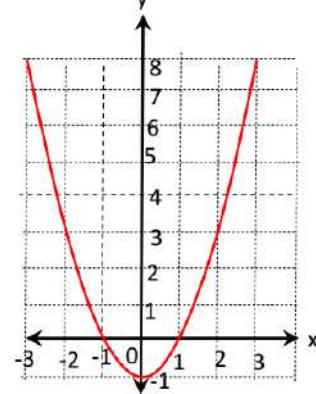
(5)



Domain =

Range =

(6)



Domain =

Range =

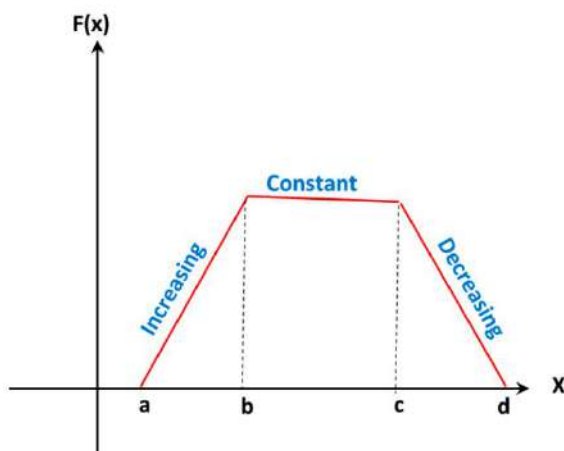
The monotony of the function

In the shown figure:

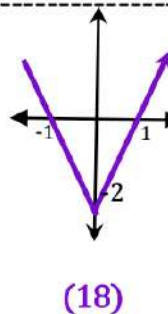
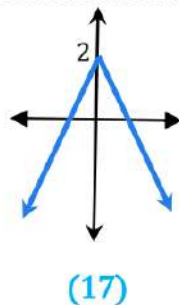
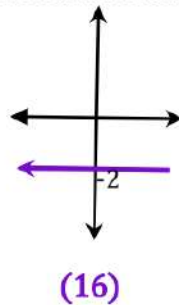
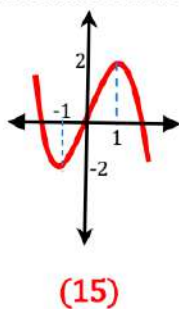
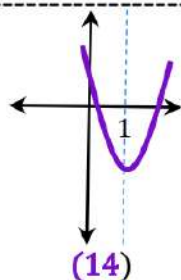
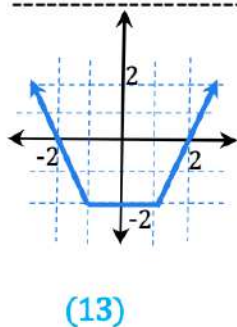
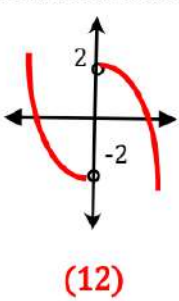
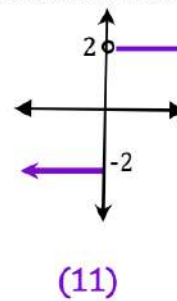
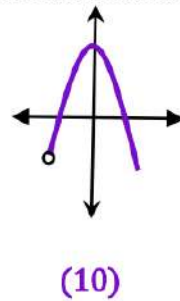
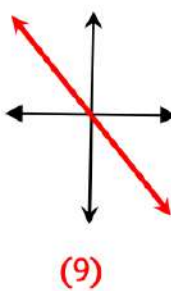
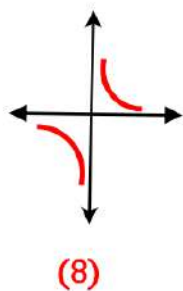
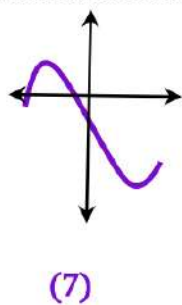
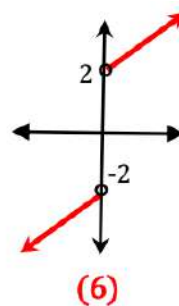
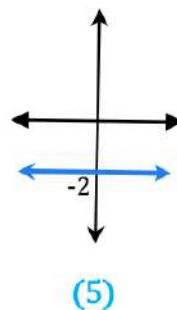
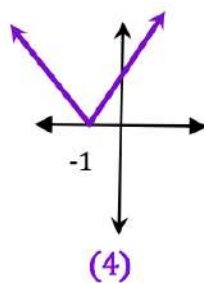
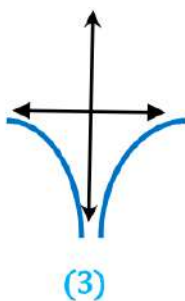
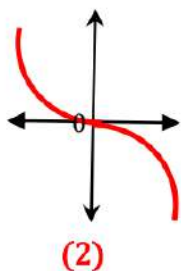
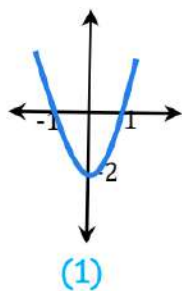
When $x \in [a, b]$ $F(x)$ increasing

When $x \in [b, c]$ $F(x)$ constant

When $x \in [c, d]$ $F(x)$ decreasing



In the following figures, deduce the range, the monotonicity



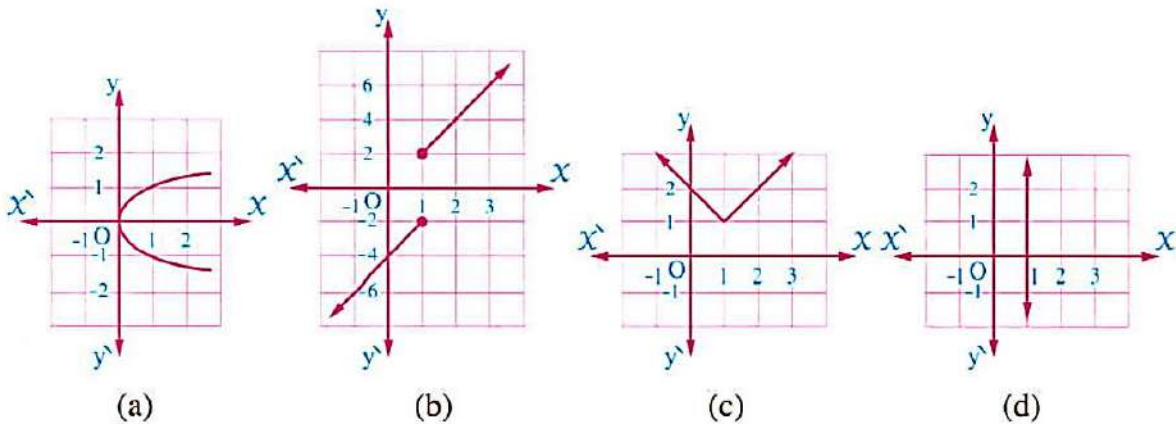
Exercises (1)

Choose the correct answer:

1)	In all the following relations , y is a function in X except	
	(a) $y = 3X + 1$ (b) $y = X^2 - 4$ (c) $X = y^2 - 2$ (d) $y = \sin X$	
2)	In all the following relations , y is a function in X except the relation	
	(a) $y = \cos X$ (b) $y = 2$ (c) $y = X^2 - 1$ (d) $y^2 = X^2 + 1$	
3)	If $f(\sqrt{X}) = X^2$, then $f(2) = \dots\dots\dots$	
	(a) $\sqrt{2}$ (b) 2 (c) 4 (d) 16	
4)	The domain of the function $f : f(X) = 5$ is	
	(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{5\}$ (d) $\{0, 5\}$	
5)	The domain of the function $f : f(X) = \frac{2X+1}{X-2}$ is	
	(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-\frac{1}{2}\}$ (c) $\mathbb{R} - \{-\frac{1}{2}, 2\}$ (d) $\mathbb{R} - \{2\}$	
6)	The domain of the function $f : f(X) = \frac{X^2+1}{X^2+4X}$ is	
	(a) $\mathbb{R} - \{1, -1\}$ (b) $\mathbb{R} - \{0, -4\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{0, 4\}$	
7)	The domain of the function f where $f(X) = \frac{5+2X}{X^2+X+1}$ is	
	(a) $\mathbb{R}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{-2, -5\}$	
8)	The domain of the function f where $f : \mathbb{R}^+ \longrightarrow \mathbb{R}$, $f(X) = \frac{X-1}{4X}$ is	
	(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R} - \{1\}$	
9)	 If the domain of the function $f : f(X) = \frac{2}{X^2-6X+k}$ is $\mathbb{R} - \{3\}$, then $k = \dots\dots\dots$	
	(a) 3 (b) 9 (c) ± 9 (d) 18	

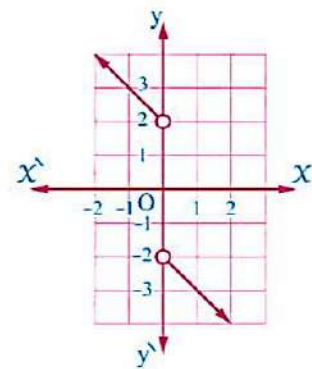
10)	 The domain of the function $f : f(x) = \sqrt{x-3}$ is (a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $[3, \infty[$ (d) $]-\infty, 3[$	
11)	The domain of the function f where $f(x) = \sqrt{4-x}$ is (a) $[4, \infty[$ (b) $]-\infty, 4[$ (c) $]4, \infty[$ (d) $]-\infty, 4]$	
12)	 The domain of the function $f : f(x) = \sqrt[3]{x-5}$ is (a) $[5, \infty[$ (b) $]-\infty, 5[$ (c) $\mathbb{R}$ (d) $\mathbb{R}^+$	
13)	 The domain of the function $f : f(x) = \frac{5}{\sqrt{x-4}}$ is (a) $[4, \infty[$ (b) $]4, \infty[$ (c) $]-\infty, 4]$ (d) $]-\infty, 4[$	
14)	The domain of the function $f : f(x) = \frac{1}{\sqrt{9-x^2}}$ is (a) $\mathbb{R}$ (b) $\mathbb{R} - [-3, 3]$ (c) $\mathbb{R} - \{-3, 3\}$ (d) $]-3, 3[$	
15)	 The domain of the function f where $f(x) = \frac{1}{\sqrt[3]{x^2-5x-6}}$ is (a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{6\}$ (c) $\mathbb{R} - \{1, -6\}$ (d) $\mathbb{R} - \{-1, 6\}$	
16)	If the domain of the function $f : f(x) = \frac{1}{\sqrt{x^2+a}}$ is $\mathbb{R} - [-5, 5]$, then $a = \dots\dots\dots$ (a) 5 (b) -25 (c) -5 (d) 25	
17)	The domain of the function f where $f(x) = \begin{cases} -2 & , & x < 2 \\ 3 & , & x > 2 \end{cases}$ is (a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{2\}$	
18)	The range of the function $f : f(x) = \begin{cases} 0 & , & x \leq 0 \\ 1 & , & x > 0 \end{cases}$ is (a) $\{1\}$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\{0, 1\}$	

- 19) The figure which represents y as a function in X is



- 20) The opposite figure represents function of X , its domain is

- (a) $\mathbb{R}$ (b) $\mathbb{R} -]-2, 2[$
(c) $\mathbb{R} - [-2, 2]$ (d) $\mathbb{R} - \{0\}$



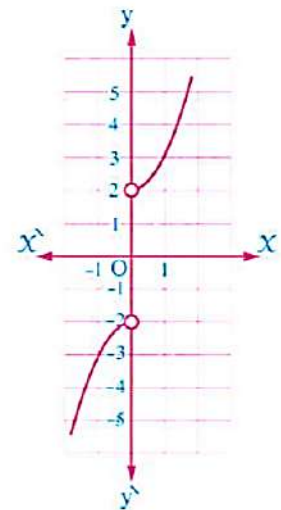
- 21) In the opposite figure :

First : The range of the function is

- (a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - [-2, 2]$
(c) $\mathbb{R}$ (d) $[-2, 2]$

Second : The function is increasing in

- (a) $]-\infty, 0[$ only
(b) $]0, \infty[$ only
(c) $]-\infty, 0[,]0, \infty[$
(d) $\mathbb{R} - [-2, 2]$



Lesson (2): Operations on function – composition of functions.

If F_1, F_2 two functions and their domains are D_1, D_2 respectively, then:

$$[1] (F_1 \pm F_2)(x) = F_1(x) \pm F_2(x) \quad , \quad D(F_1 \pm F_2) = D_1 \cap D_2$$

$$[2] (F_1 \times F_2)(x) = F_1(x) \times F_2(x) \quad , \quad D(F_1 \times F_2) = D_1 \cap D_2$$

$$[3] \left(\frac{F_1(x)}{F_2(x)} \right)(x) = \frac{F_1(x)}{F_2(x)} \text{ Where } F_2(x) \neq 0,$$

$$D\left(\frac{F_1}{F_2}\right) = D_1 \cap D_2 - Z_2 \text{ where } Z_2 \text{ is the set of zeroes of } F_2$$

Examples

[1] If $F: \mathbb{R}^+ \rightarrow \mathbb{R}$ where $F(x) = 2x^2 + 1$,

$G:]-\infty, 4] \rightarrow \mathbb{R}$ where $G(x) = x - 4$. Find:

$$(1) (F + G)(x) \quad (2) (F - G)(x) \quad (3) (F \times G)(x) \quad (4) \frac{F}{G}(x)$$

and find the domain of each, then find: $(F + G)(3), (F - G)(0), (F \times G)(-2), \frac{F}{G}(1)$

Sol.

$$(1) (F + G)(x) = (2x^2 + 1) + (x - 4) \\ = 2x^2 + x - 3$$

$$\text{Domain of } (F + G)(x) = D_1 \cap D_2 \\ = \mathbb{R}^+ \cap]-\infty, 4] =]0, 4]$$

$$(F + G)(3) = 2(3)^2 + (3) - 4 = 17$$

$$(3) (F \times G)(x) = (2x^2 + 1)(x - 4) \\ = 2x^3 - 8x^2 + x - 4$$

$$\text{Domain of } (F + G)(x) = D_1 \cap D_2 \\ = \mathbb{R}^+ \cap]-\infty, 4] =]0, 4]$$

$$(F \times G)(-2) = 2(-2)^3 - 8(-2)^2 + (-2) = -54$$

$$(2) (F - G)(x) = (2x^2 + 1) - (x - 4) \\ = 2x^2 - x + 5$$

$$\text{Domain of } (F + G)(x) = D_1 \cap D_2 \\ = \mathbb{R}^+ \cap]-\infty, 4] =]0, 4]$$

$$(F - G)(0) = 2(0)^2 - (0) + 5 = 5$$

$$(4) \frac{F}{G}(x) = \frac{2x^2 + 1}{x - 4}$$

$$\text{Domain of } \frac{F}{G}(x) = D_1 \cap D_2 - \{Z_2\}$$

Where Z_2 is the set of zeroes of F_2

$$\text{To get } Z_2: x - 4 = 0 \quad x = 4 \quad Z_2 = \{4\}$$

$$\therefore \text{Domain of } \frac{F}{G}(x) = \mathbb{R}^+ \cap]-\infty, 4] - \{4\} \\ =]-\infty, 4[\quad , \quad \frac{F}{G}(1) = \frac{2(1)^2 + 1}{1 - 4} = \frac{3}{-3} = -1$$

[2] If F, G are two function where $F(x) = \sqrt{x - 2}$, $G(x) = \sqrt{4 - x}$, find:

- (1) $(F + G)(x)$ (2) $(F - G)(x)$ (3) $(F \times G)(x)$ (4) $\frac{F}{G}(x)$

And find the domain of each of them.

Sol.

* The composition of functions

[3] If $F(x) = x^2$, $G(x) = x+1$. Find:

(1) $(F \circ G)(1)$

(2) $(G \circ F)(1)$

(3) $(G \circ G)(-1)$

Sol.

$$(1) (F \circ G)(1) = F[G(1)] = F[1+1] = F(2) = (2)^2 = 4$$

$$(2) (G \circ F)(1) = G[F(1)] = G[(1)^2] = G(1) = 1+1 = 2$$

$$(3) (G \circ G)(-1) = G[G(-1)] = G[-1+1] = G(0) = 0+1 = 1$$

[4] If $F(x) = 2x + 1$, $G(x) = x^2 - 3$. Find:

(1) $(F \circ G)(x)$

(2) $(G \circ F)(x)$

(3) $(G \circ G)(x)$

Sol.

Note that:

$$[1] (F \circ G)(x) \neq (G \circ F)(x)$$

$$[2] \text{ To determine the domain of } (F \circ G)(x) = \{x: x \in \text{domain of } G, G \in \text{domain of } F\}$$

$$(i) \text{ Find } D_1 = \text{domain of } G$$

$$(ii) \text{ Find } D_2 = \text{value of } x \text{ that make } G \text{ in the domain of } F$$

$$(iii) \text{ Find } D_1 \cap D_2 \text{ which is the domain of } (F \circ G)(x)$$

[5] Find the domain of $(F \circ G)$, if $F(x) = \frac{4}{x-1}$, $G(x) = \frac{3}{x+3}$

Sol.

[6] Find the domain of $(F \circ G)$, if $F(x) = \sqrt{x-2}$, $G(x) = \sqrt{5-x}$

Sol.

Exercises (2)

Choose the correct answer:

1)	If $f(x) = \sqrt[3]{x-1}$, $g(x) = \sqrt{x-1}$, then the domain of $(f-g)$ is (a) $[-1, \infty[$ (b) $[1, \infty[$ (c) $\mathbb{R}$ (d) $]-\infty, 1]$
2)	The domain of the function $f : f(x) = \sqrt{x-1} + \sqrt{x+2}$ is (a) $[-2, \infty[$ (b) $[1, \infty[$ (c) $[1, \infty[$ (d) $]-2, \infty[$
3)	The domain of the function $f : f(x) = \sqrt{x+2} - \sqrt{3-x}$ is (a) $\mathbb{R}$ (b) $\mathbb{R} - [-2, 3]$ (c) $[-2, 3]$ (d) $]-2, 3[$
4)	If $f, g : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = 3x + 1$, and $(f+g)(x) = x^3 + 2x - 1$, then $g(-1) = \dots\dots\dots$ (a) -2 (b) -1 (c) zero (d) 2

5)	If f is a function where $f = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$ and g is a function where $g = \{(1, 3), (3, 5), (5, 7)\}$, then $f + g = \dots\dots\dots$ (a) $\{(1, 2), (2, 4), (3, 6), (4, 8), (1, 3), (3, 5), (5, 7)\}$ (b) $\{(1, 2), (2, 4), (3, 6), (4, 8), (5, 7)\}$ (c) $\{(1, 5), (3, 11)\}$ (d) $\{(1, 5), (2, 4), (3, 11), (4, 8), (5, 7)\}$
6)	If $f: \mathbb{R}^+ \longrightarrow \mathbb{R}$ where $f(x) = x^2 - 7$, $g: [-3, 2] \longrightarrow \mathbb{R}$ where $g(x) = x^2 + 3$, then $(f - g)(-5) = \dots\dots\dots$ (a) -5 (b) -10 (c) -20 (d) undefined.
7)	The domain of the function $f: f(x) = \sqrt{x-4} \sqrt[3]{x-2}$ is $\dots\dots\dots$ (a) $]4, \infty[$ (b) $[2, \infty[$ (c) $\mathbb{R}$ (d) $[4, \infty[$
8)	If $f: \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = x^2 - x$, $g: \mathbb{R} \longrightarrow \mathbb{R}$ where $g(x) = 3x - 2$, then $(f \times g - 6g)(2) = \dots\dots\dots$ (a) 24 (b) 40 (c) 16 (d) -16
9)	If f, g are two real functions where $f(x) = \frac{x-2}{x^2-3x+2}$, $g(x) = x-3$, then $\left(\frac{f}{g}\right)(3) = \dots\dots\dots$ (a) $\frac{1}{2}$ (b) 1 (c) zero (d) undefined.
10)	The domain of the function $f: f(x) = \frac{x-5}{\sqrt{2x-3}}$ is $\dots\dots\dots$ (a) $\mathbb{R} - \{5\}$ (b) $] \frac{3}{2}, \infty[$ (c) $[\frac{3}{2}, \infty[- \{5\}$ (d) $\mathbb{R} - \{\frac{3}{2}\}$
11)	The domain of the function $f: f(x) = \frac{\sqrt{x-2}}{x-3}$ is $\dots\dots\dots$ (a) $\mathbb{R}$ (b) $\{3\}$ (c) $[2, \infty[$ (d) $[2, \infty[- \{3\}$

12)	The domain of the function f where $f(x) = \frac{x+1}{\sqrt[3]{x-1}}$ is (a) $\mathbb{R} - \{1\}$ (b) $\mathbb{R} - \{-1\}$ (c) $[-1, \infty[$ (d) $\mathbb{R}$
13)	The domain of the function f where $f(x) = \frac{x+1}{\sqrt[3]{x-1}}$ is (a) $\mathbb{R} - \{1\}$ (b) $\mathbb{R} - \{-1\}$ (c) $[-1, \infty[$ (d) $\mathbb{R}$
14)	The domain of the function $f : f(x) = \frac{\sqrt{x-4}}{(x-3)(x-5)}$ is (a) $\mathbb{R}$ (b) $\mathbb{R} - \{3, 5\}$ (c) $[4, \infty[$ (d) $[4, \infty[- \{5\}$
15)	If $f(x) = \sqrt{x+5}$, $g(x) = x^2$, then $(f \circ g)(2) = \dots\dots\dots$ (a) 7 (b) 3 (c) 4 (d) 9
16)	If $f(1) = 4$, $g(4) = 7$, then $(g \circ f)(1) = \dots\dots\dots$ (a) 1 (b) 4 (c) 7 (d) 11
17)	If $f : [-2, 3] \longrightarrow \mathbb{R}$ where $f(x) = x + 1$, $g : [0, 2] \longrightarrow \mathbb{R}$ where $g(x) = x^2$, then $(f \circ g)(2) = \dots\dots\dots$ (a) 4 (b) -1 (c) 5 (d) undefined.
18)	If $f(x) = 5x + 4$, $g(x) = 2 - x$, then $(f \circ g)(-2) + (g \circ f)(5) = \dots\dots\dots$ (a) 24 (b) -27 (c) 49 (d) -3
19)	If $f(x) = \sqrt[3]{x}$, $g(x) = x^3$, then $(f \circ g)(x) = \dots\dots\dots$ (a) x (b) x^3 (c) $\sqrt[3]{x}$ (d) x

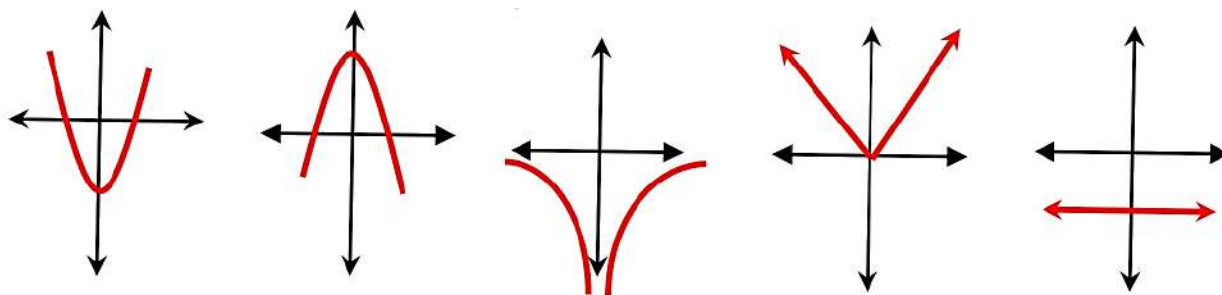
20)	If $f(x) = x - 3$, $g(x) = x^2$, then $(f \circ g)(x) = \dots\dots\dots$ (a) $(x - 3)^2$ (b) $x^2 - 3$ (c) $x^2 + 3$ (d) $\sqrt{x - 3}$
21)	If $f(x) = 2x + 5$ and $g(x) = x^2$, then $f(g(x)) = \dots\dots\dots$ (a) $(2x + 5)^2$ (b) $2x^2 + 5$ (c) $(2x^2 + 5)^2$ (d) $2x^2 + 10$
22)	If $f(x) = ax + 3$, $g(x) = 2x + a$ and $(f \circ g)(2) = 15$, then $a \in \dots\dots\dots$ (a) $\{2, 6\}$ (b) $\{-2, -6\}$ (c) $\{-6, 2\}$ (d) $\{-2, 6\}$
23)	If $f(x) = \frac{1}{x}$, $g(x) = x^2 - 1$, then the domain of $(f \circ g) = \dots\dots\dots$ (a) $\{0, 1, -1\}$ (b) $\mathbb{R} - \{0, 1, -1\}$ (c) $\mathbb{R} - \{1, -1\}$ (d) $[-1, 1]$
24)	If $f(x) = \sqrt{x}$, $g(x) = x^2$, then the domain of $(f \circ g) = \dots\dots\dots$ (a) $]-\infty, 0]$ (b) $[0, \infty[$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}$
25)	The domain of the function $(f \circ g)$ where : $f(x) = \sqrt{x - 2}$, $g(x) = \sqrt{6 - x}$ is $\dots\dots\dots$ (a) $\mathbb{R}$ (b) $[2, \infty[$ (c) $]-\infty, 2]$ (d) $]-\infty, 2[$

Lesson (3): Some properties of functions

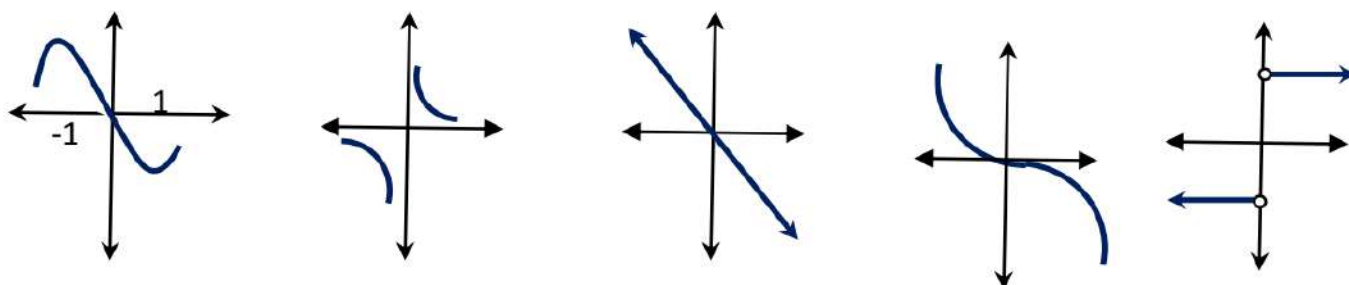
(Even, odd – one-to-one functions)

First: Graphically

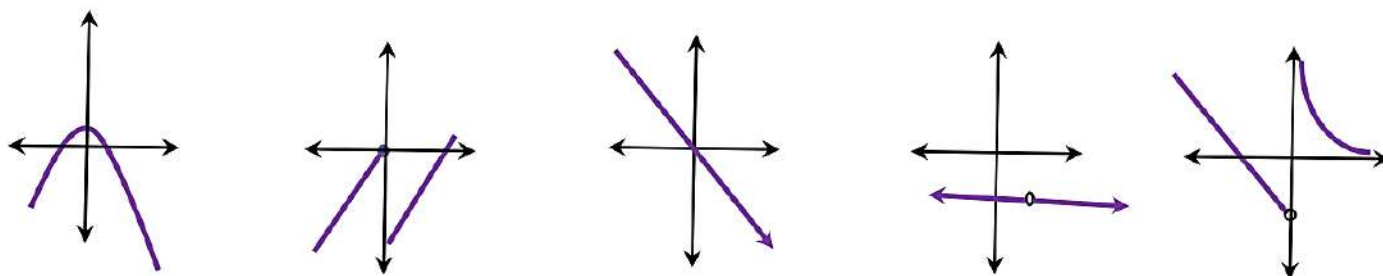
[1] If the curve of the function $F(x)$ is symmetric about Y-axis, then $F(x)$ is even function.



[2] If the curve of the function $F(x)$ is symmetric about the origin point, then $F(x)$ is odd function.



[3] Otherwise, the function is neither even nor odd



Second: Algebraically:

[1] If $F(-x) = F(x)$, then $F(x)$ is even Function

Such as: $F(x) = x^{\text{even}}$, $F(x) = a$ (Constant), $F(x) = \cos x$, $F(x) = |x|$,

[2] If $F(-x) = -F(x)$, then $F(x)$ is odd Function

Such as: $F(x) = x^{\text{odd}}$, $F(x) = \sin x$, $F(x) = \tan x$,

Examples

Which of the following functions is even, odd, or otherwise:

(1) $F(x) = x^2 - 3$

The domain of F is $\mathbb{R}$

$\therefore$ for each x , $-x \in \mathbb{R}$, then

$$F(-x) = (-x)^2 - 3 = x^2 - 3 = F(x)$$

$\therefore F(x)$ is even

(2) $F(x) = \sin x - x^3$

The domain of F is $\mathbb{R}$

$\therefore$ for each x , $-x \in \mathbb{R}$, then

$$F(-x) = \sin(-x) - (-x)^3 = -\sin x + x^3 = -F(x)$$

$\therefore F(x)$ is odd

(3) $F(x) = x^2 - \cos x$

The domain of F is $\mathbb{R}$

$\therefore$ for each x , $-x \in \mathbb{R}$, then

$$F(-x) = (-x)^2 - \cos(-x) = x^2 - \cos x = F(x)$$

$\therefore F(x)$ is even

(4) $F(x) = 5^x - 5^{-x}$

The domain of F is $\mathbb{R}$

$\therefore$ for each x , $-x \in \mathbb{R}$, then

$$F(-x) = 5^{-x} - 5^x = -F(x)$$

$\therefore F(x)$ is odd

(5) $F(x) = x^3 \tan x + \cos x$

for each x , $-x \in \text{domain of } F$, then

$$F(-x) = (-x)^3 \tan(-x) + \cos(-x) = x^3 \tan x + \cos x = F(x)$$

$\therefore F(x)$ is even

(6) $F(x) = x^3 - x^2$

$\therefore$ for each x , $-x \in \mathbb{R}$, then

$$F(-x) = (-x)^3 - (-x)^2 = -x^3 - x^2$$

$\therefore F(x)$ is neither even nor odd (NENO)

(7) $F(x) = \sec x (\sin x + \tan x)$

$$F(-x) = \sec(-x) [\sin(-x) + \tan(-x)]$$

$$= \sec x [-\sin x - \tan x]$$

$$= -\sec x [\sin x + \tan x] = -F(x)$$

$\therefore F(x)$ is odd

(8) $F(x) = \frac{5}{3^x} + 5 \times 3^x$

$$F(x) = \frac{5}{3^{-x}} + 5 \times 3^{-x} = 5 \times 3^x + \frac{5}{3^x}$$

$$= F(x)$$

$\therefore F(x)$ is even

$$(9) F(x) = \frac{\sin x}{x^2 - \tan x}$$

$$F(-x) = \frac{\sin(-x)}{(-x)^2 - \tan(-x)} = \frac{-\sin x}{x^2 + \tan x}$$

∴ F(x) is neither even nor odd (NENO)

$$(11) F(x) = \frac{|x|}{\tan x - \sin x}$$

$$F(-x) = \frac{|-x|}{\tan(-x) - \sin(-x)} = \frac{|x|}{-\tan x - \sin x} = -F(x)$$

∴ F(x) is odd

$$(13) F(x) = \sqrt{x-2}$$

The domain of F is the set of values of x satisfying $x-2 \geq 0$ i.e $x \geq 2$

∴ the domain of F = $[2, \infty[$

∴ $-x \notin [2, \infty[$

∴ F(x) is neither even nor odd (NENO)

$$(15) F(x) = \frac{x^3 \csc 3x}{1+x^4}$$

$$(10) F(x) = \left(x - \frac{1}{x}\right)^3$$

$$F(-x) = \left(-x - \frac{1}{-x}\right)^3 = -\left(x - \frac{1}{x}\right)^3 = -F(x)$$

∴ F(x) is odd

$$(12) f(x) = \left(\frac{x-1}{x+1}\right)^5 - \left(\frac{x+1}{x-1}\right)^5$$

$$F(-x) = \left(\frac{-x-1}{-x+1}\right)^5 - \left(\frac{-x+1}{-x-1}\right)^5 =$$

$$- \left(\frac{x+1}{x-1}\right)^5 - \left(\frac{x-1}{x+1}\right)^5 = -F(x)$$

∴ F(x) is odd

$$(14) F(x) = x^2 + 5, x \in [-3, 3[$$

-3 ∈ the domain of F but 3 ∉ the domain of F

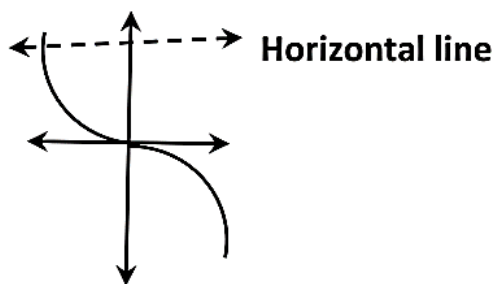
∴ F(x) is neither even nor odd (NENO)

$$(16) F(x) = \frac{3x^3 - \sec x}{x^5 - 4x}$$

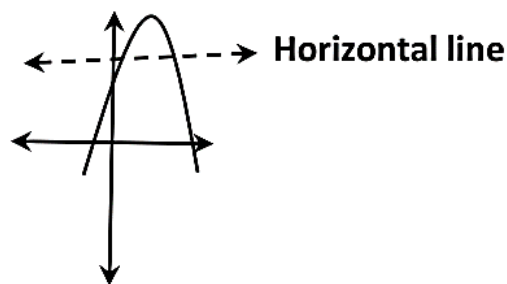
* The one to one function

The function $F: x \longrightarrow y$ is called one to one function if:

for every $a, b \in$ the domain of F and $F(a) = F(b)$, then $a = b$

First: Graphically

The horizontal line cuts the curve in one point, then the function is mono function



The horizontal line cuts the curve in two points, then the function is not mono

Second: Algebraically:Examples

Prove that the following function is one to one function:

[1] $F(x) = 3x - 4$

[2] $F(x) = x^3 - 2$

[3] $F(x) = \frac{4}{3x+1}$

Sol.

[1] Let $a, b \in$ the domain of F

$$\because F(a) = 3a - 4, F(b) = 3b - 4 \text{ and let } F(a) = F(b)$$

$$\therefore 3a - 4 = 3b - 4$$

$$\therefore 3a = 3b$$

$$\therefore a = b$$

$\therefore$ The function F is one to one function.

[2] Let $a, b \in$ the domain of F

$$\because F(a) = a^3 - 2, F(b) = b^3 - 2 \text{ and let } F(a) = F(b)$$

$$\therefore a^3 - 2 = b^3 - 2$$

$$\therefore a^3 = b^3$$

$$\therefore a = b$$

$\therefore$ The function F is one to one function.

[3] Let $a, b \in$ the domain of F

$$\therefore F(a) = \frac{4}{3a+1}, F(b) = \frac{4}{3b+1} \text{ and let } F(a) = F(b)$$

$$\therefore \frac{4}{3a+1} = \frac{4}{3b+1} \quad \therefore 3a+1 = 3b+1 \quad \therefore 3a = 3b \quad \therefore a = b$$

$\therefore$ The function F is one to one function.

Prove that the following two functions is not one to one function:

[4] $F(x) = 2 - x^2$

[5] $F(x) = x^2 + x$

Sol.

Exercises (3)

Choose the correct answer:

1)	The even function from the functions that are defined by the following rules is	
	(a) $f(x) = x^3$ (b) $f(x) = \sin x$	
	(c) $f(x) = x \cos x$ (d) $f(x) = x \sin x$	
2)	The odd function from the functions that are defined by the following rules is	
	(a) $f(x) = x^2 \sin x$ (b) $f(x) = \tan^2 x$	
	(c) $f(x) = \cos x$ (d) $f(x) = 1$	
3)	The type of the function $f : f(x) = \frac{\sin x}{x}$ is	
	(a) even. (b) odd.	
	(c) neither even nor odd. (d) one - to - one.	

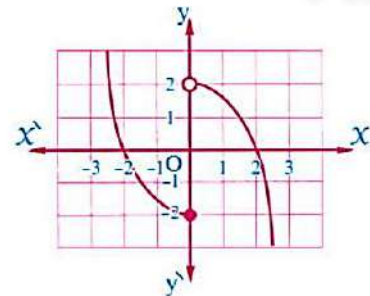
4)	The function $f : f(x) = x \cos x$ is (a) even. (b) odd. (c) neither even nor odd. (d) one - to - one.	
5)	Each of the following is a rule of even function except (a) $f(x) = \sin x$ (b) $f(x) = \cos x$ (c) $f(x) = x^2 - 1$ (d) $f(x) = 1$	
6)	Which of the following rules does not represent an even function ? (a) $y = \frac{1}{x^2}$ (b) $y = \sec x$ (c) $y = x^2 + \sin x$ (d) $y = 3x^4 - 2x^2 + 27$	
7)	If f is an odd function , $f(1) = 2$, then which of the following points lies on the curve of f ? (a) $(-1, 2)$ (b) $(-1, -2)$ (c) $(1, -2)$ (d) $(-1, 0)$	
8)	If f is an odd function , $a \in$ the domain of f , then $f(a) + f(-a) = \dots\dots\dots$ (a) zero. (b) $2f(a)$ (c) $2a$ (d) $f(a)$	
9)	If f is an even function , then $f(a) - f(-a) = \dots\dots\dots$ (a) zero. (b) $f(a)$ (c) $2f(a)$ (d) $f(2a)$	
10)	The function $f : f(x) = (x + \sin x) (\dots\dots\dots)$ is even. (a) $x^3 + \tan x$ (b) $1 + \sin x$ (c) $4 - x^2$ (d) $x - \cos x$	
11)	If f is an even function and the curve of the function passes through point $(-3, 2m + 1)$, and $f(3) = 5$, then $m = \dots\dots\dots$ (a) -1 (b) zero (c) 1 (d) 2	
12)	 If the function f is an even over $[a, b]$, then $b = \dots\dots\dots$ (a) a (b) $-a$ (c) $2a$ (d) a^3	
13)	If f is an even function and $f(5) = 1$, $f(-5) = 3 - k$, then $k = \dots\dots\dots$ (a) 1 (b) 5 (c) 3 (d) 2	

14)	The one - to - one function from those defined by the following rules is (a) $f(x) = 3 - x^2$ (b) $f(x) = x^3 + 3$ (c) $f(x) = \sin x \tan x$ (d) $f(x) = x^2 + x$	
15)	The functions defined by the following rules are one - to - one except (a) $f(x) = x^3$ (b) $g(x) = 3x$ (c) $h(x) = \frac{1}{x}$ (d) $n(x) = x^2$	
16)	All functions defined by the following rules are not one - to - one except (a) $f(x) = x^2$ (b) $f(x) = 5$ (c) $f(x) = x + 2$ (d) $f(x) = \sin x$	
17)	If f, g are two functions where $f(x) = x^3$, $g(x) = x + 2$, then $(g \circ f)$ is function. (a) a one - to - one (b) an odd (c) an even (d) a linear	
18)	The function $f : f(x) = x^3 + 5x$ is symmetric about (a) the x-axis. (b) the y-axis. (c) the origin. (d) can not be determined.	
19)	The function $f : f(x) = x^2 + x^4 + 1$ is symmetric about (a) the origin. (b) the x-axis. (c) the y-axis. (d) it has neither symmetric point nor symmetric line.	
20)	The opposite figure represents the curve of the function f, then f is (a) one - to - one. (b) an even function. (c) an odd function. (d) neither odd nor even.	

21)

The opposite figure represents the curve of the function f , then f is

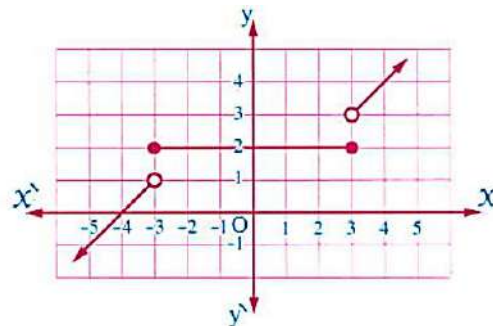
- (a) one - to - one. (b) an even function.
(c) an odd function. (d) neither odd nor even.



22)

The opposite figure represents the curve of the function f , then f is

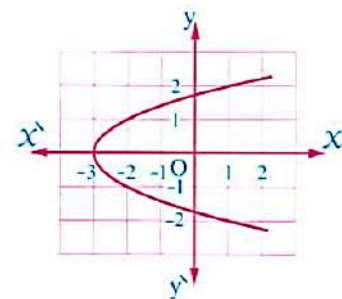
- (a) one - to - one function.
(b) an even function.
(c) an odd function.
(d) neither odd nor even.



23)

 The curve represented in the opposite figure is symmetric about the straight line whose equation is

- (a) $x = 0$ (b) $y = 0$
(c) $y = -2$ (d) $x = 2$



Lesson (4): Graphical representation of functions of basic functions

$$F(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_n x^n$$

is called a polynomial function of n degree,

where n is the highest power of the independent variable x.

where $a_0, a_1, a_2, a_3, \dots, a_n \in \mathbb{R}$, $n \neq 0$, $n \in \mathbb{N}$

Notice:

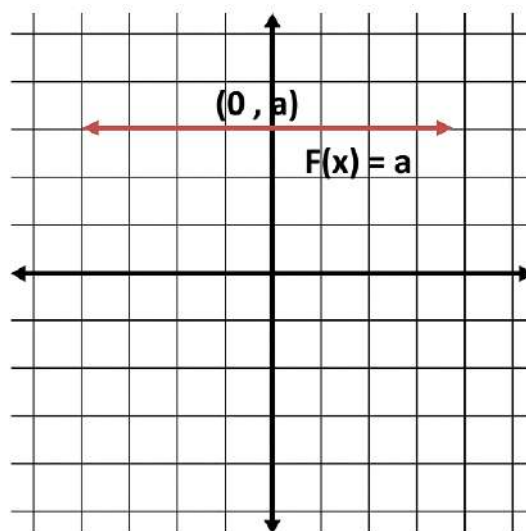
- 1- $F(x) = a_0$, $a_0 \neq 0$ is called a constant polynomial function
- 2- Polynomial functions of first degree are called linear functions, second degree are called quadratic functions, and the third degree are called cubic functions.
- 3- Zeros of the polynomial function are the x-coordinates of the point(s) of the intersection of the curve with the X-axis.
- 4- Two polynomial functions f and g are equal if they have the same degree and the coefficients of corresponding powers of x are equal.

The original functions

First: The constant function:

The function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = a$, $a \in \mathbb{R}$ is called constant function, represented by a straight line parallel to the x-axis and cuts the y-axis at the point $(0, a)$.

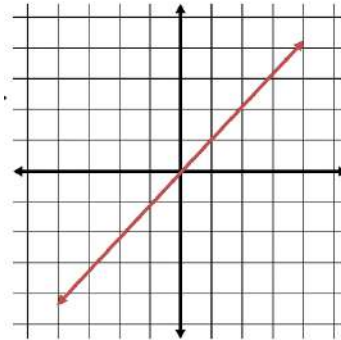
- 1) **Domain** = $\mathbb{R}$
- 2) **Range** = $\{a\}$
- 3) **Mont. : Contant on $\mathbb{R}$**
- 4) **Type** : even
- 5) **Not one-to-one**



Second: The linear function:

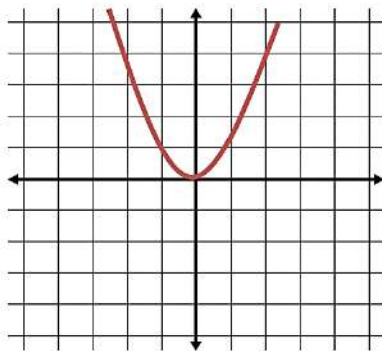
The function $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x$ is called a linear function, represented by a st. line passing through the origin point.

- 1) **Domain** = $\mathbb{R}$
- 2) **Range** = $\mathbb{R}$
- 3) **Mont.** :increasing on $\mathbb{R}$
- 4) Type: odd
- 5) one-to-one

**Third: The quadratic function: (Second degree)**

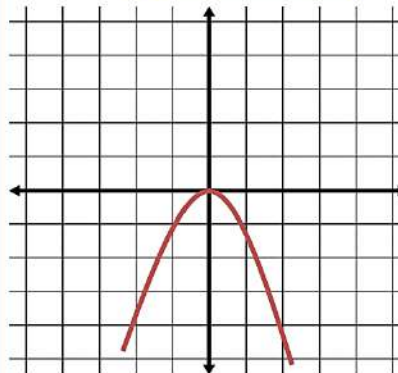
$\therefore f(x) = (x + a)^2 + b, \therefore \text{vertex} = (-a, b)$

$$f(x) = x^2$$



Domain = $\mathbb{R}$
Range = $[0, \infty[$
Type : even
Dec. on $]-\infty, 0[$
Inc. on $]0, \infty[$
Not One - to - one

$$f(x) = -x^2$$

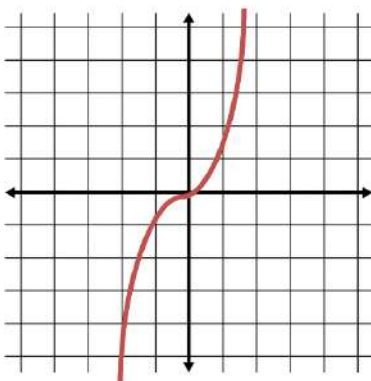


Domain = $\mathbb{R}$
Range = $]-\infty, 0[$
Type : even
Inc. on $]-\infty, 0[$
Dec. on $]0, \infty[$
Not One - to - one

Fourth: The cubic function: (Third degree)

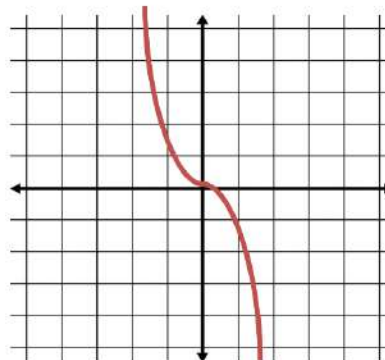
$\therefore f(x) = (x + a)^3 + b, \therefore \text{Point of symmetry} = (-a, b)$

$$f(x) = x^3$$



Domain = $\mathbb{R}$
Range = $\mathbb{R}$
Odd
One - to - one
Inc. on $\mathbb{R}$

$$f(x) = -x^3$$

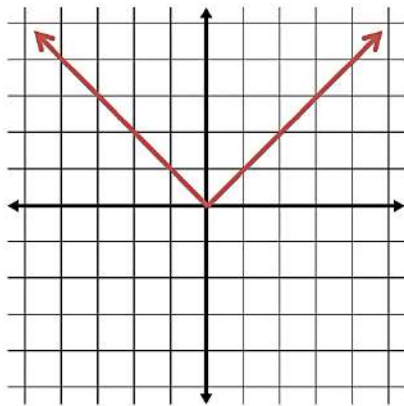


Domain = $\mathbb{R}$
Range = $\mathbb{R}$
Odd
One - to - one
Dec. on $\mathbb{R}$

Fifth: The absolute value function:

$$\therefore f(x) = |x + a| + b, \therefore \text{vertex} = (-a, b)$$

$$f(x) = |x|$$



Domain = $\mathbb{R}$

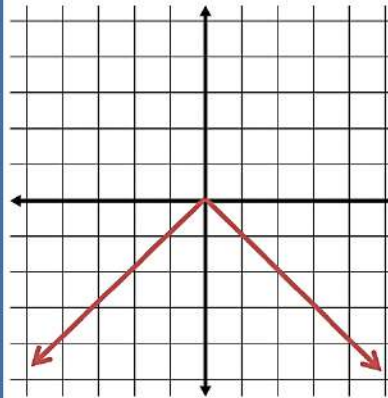
Range = $[0, \infty[$

Type : even

Dec. on $] -\infty, 0[$

Inc. on $] 0, \infty[$

$$f(x) = -|x|$$



Domain = $\mathbb{R}$

Range = $] -\infty, 0]$

Type : even

Inc. on $] -\infty, 0[$

Dec. on $] 0, \infty[$

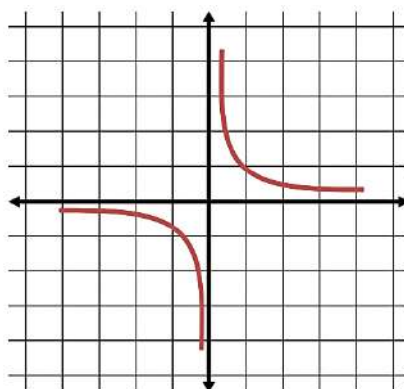
.N

Sixth: The fraction function:

$$\therefore f(x) = \frac{1}{x + a} + b,$$

$$\therefore \text{Point of symmetry} = (-a, b)$$

$$f(x) = \frac{1}{x}$$



Domain = $\mathbb{R} - \{0\}$

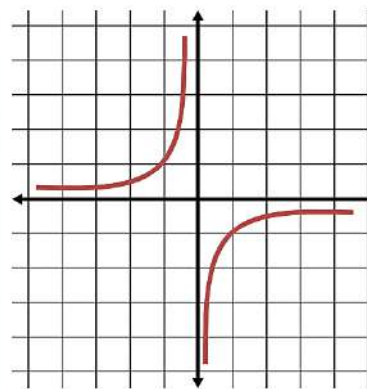
Range = $\mathbb{R} - \{0\}$

Type : Odd

One - to - one

Dec. on its domain

$$f(x) = -\frac{1}{x}$$



Domain = $\mathbb{R} - \{0\}$

Range = $\mathbb{R} - \{0\}$

Type : Odd

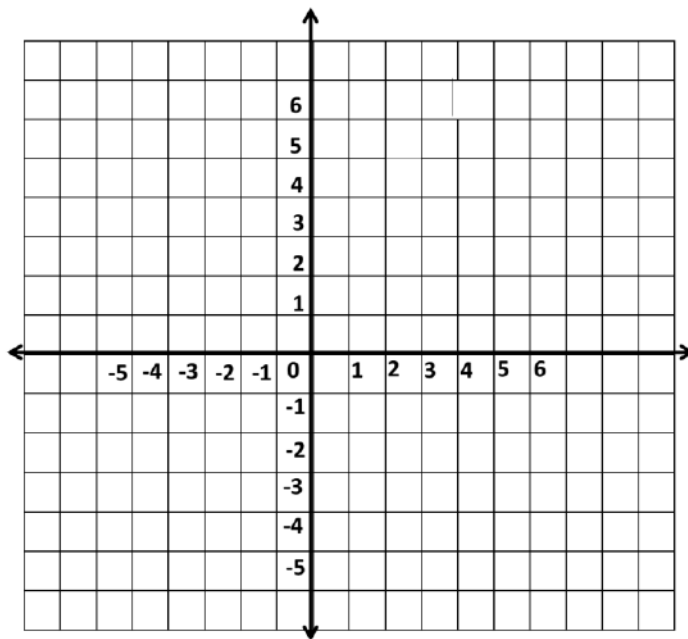
One - to - one

Inc. on its domain

Graph each of the following functions, from the graph find the domain and the range and discuss its monotonicity and its type:

$$(1) \quad f(x) = \begin{cases} x & , x \leq 0 \\ x^2 & , x > 0 \end{cases}$$

Sol.



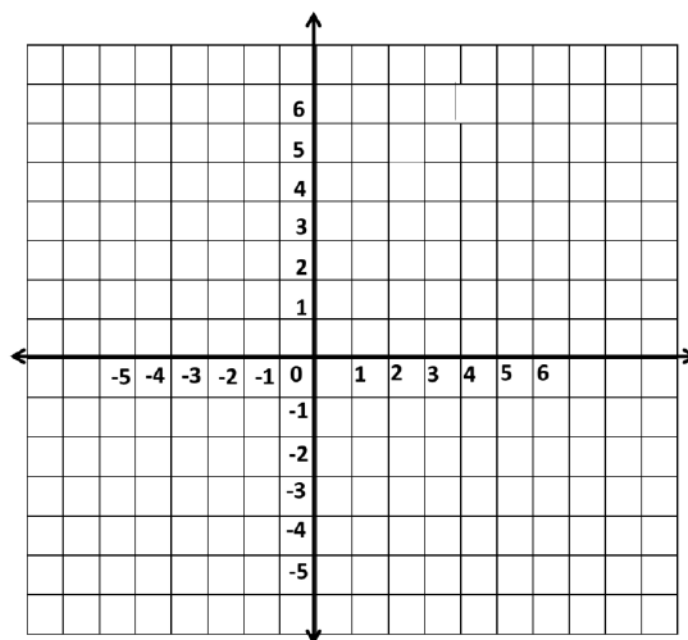
$$(2) \quad f(x) = \begin{cases} |x| & , x \leq 0 \\ \frac{1}{x} & , x > 0 \end{cases}$$

Sol.

$$f(x) = \begin{cases} -x & , x \leq 0 \\ \frac{1}{x} & , x > 0 \end{cases}$$

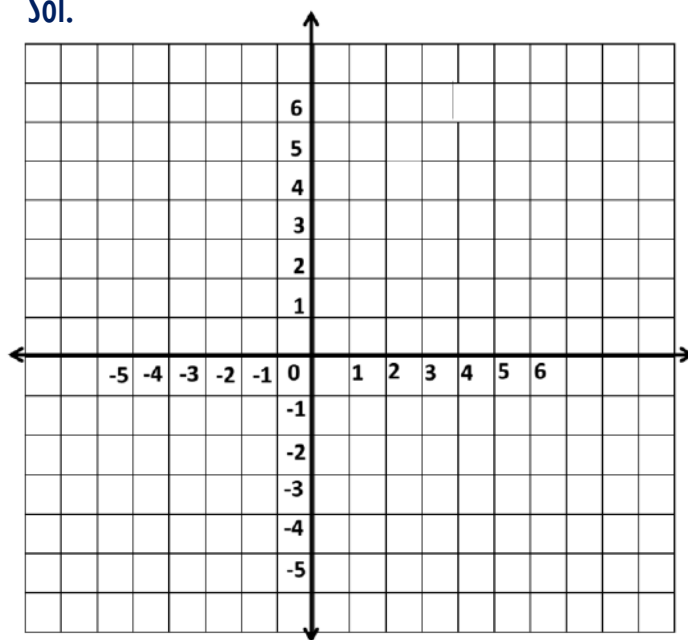
$$f(x) = -x \quad , x \leq 0$$

$$f(x) = \frac{1}{x} \quad , x > 0$$



$$(3) \quad f(x) = \begin{cases} x^3 & , x < 1 \\ 1 & , x > 1 \end{cases}$$

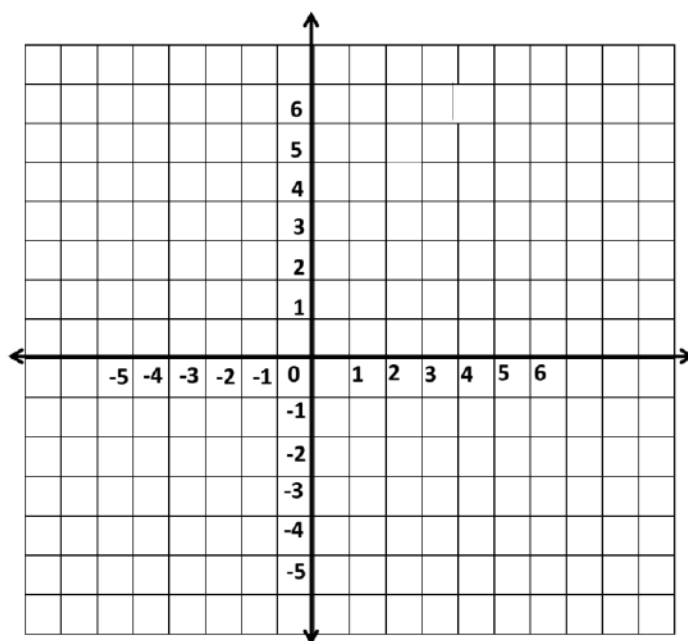
Sol.



$$(4) \quad f(x) = \frac{4 - x^2}{x + 2} \quad \text{Sol.}$$

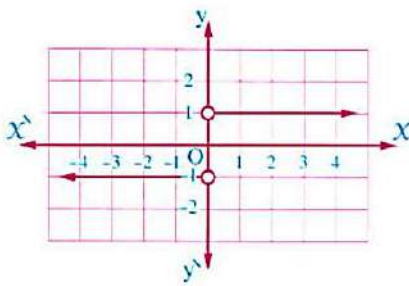
$$f(x) = \frac{-(x-2)(x+2)}{x+2} = -x + 2$$

x	-4	-3	(-2)	-1	0
y	6	5	(4)	3	2



Exercises (4)

Choose the correct answer:

1)	If $f(x) = 5$, then the domain of the function f is (a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{5\}$ (d) $\mathbb{R} - \{5\}$	
2)	If $f(x) = 7$, then the range of the function f is (a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\{7\}$ (d) $\mathbb{R} - \{7\}$	
3)	The range of the function $f : f(x) = \begin{cases} 0 & \text{when } x \leq 0 \\ 1 & \text{when } x > 0 \end{cases}$ is (a) $\{1\}$ (b) $\{0\}$ (c) $\mathbb{R}$ (d) $\{0, 1\}$	
4)	In the opposite figure : The range of the function is (a) $\{1\}$ (b) $\{1, -1\}$ (c) $\{-1\}$ (d) $\mathbb{R}$ 	
5)	The range of the function $f : f(x) = \frac{3x^2 - 3}{x^2 - 1}$ is (a) $\mathbb{R} - \{1, -1\}$ (b) $\mathbb{R} - \{3, -3\}$ (c) $\{3, -3\}$ (d) $\{3\}$	
6)	The range of the function $f : f(x) = \frac{2x^3 - 2x}{x^2 - 1}$ is (a) $\mathbb{R} - \{1, -1\}$ (b) $\mathbb{R} - \{2, -2\}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}^+ - \{2\}$	
7)	The range of the function $f : f(x) = \begin{cases} x & , x > 0 \\ -2 & , x \leq 0 \end{cases}$ is (a) $\mathbb{R}^+$ (b) $\mathbb{R}^+ - \{-2\}$ (c) $\mathbb{R}^+ \cup \{-2\}$ (d) $\mathbb{R}$	

8)

The function f where $f(x) = \begin{cases} 2 & , \quad x > 0 \\ -2 & , \quad x < 0 \end{cases}$ is symmetric about the point

(a) (2 , 0)

(b) (-2 , 0)

(c) (0 , 0)

(d) (2 , -2)

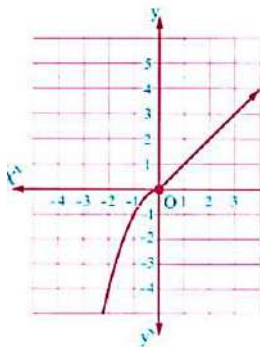
9)

The function $f : f(x) = \begin{cases} -x^2 & , \quad x < 0 \\ \frac{1}{x} & , \quad x > 0 \end{cases}$ is increasing on

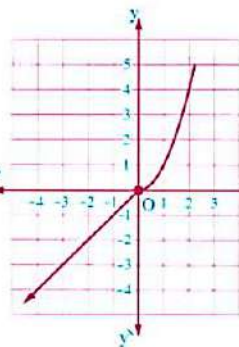
(a) $\mathbb{R}$ (b) $\mathbb{R}^-$ (c) $\mathbb{R}^+$ (d) $\mathbb{R} - \{0\}$

10)

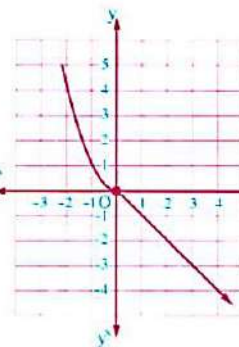
The curve of the function $f : f(x) = \begin{cases} x^2 & , \quad x > 0 \\ x & , \quad x \leq 0 \end{cases}$ is



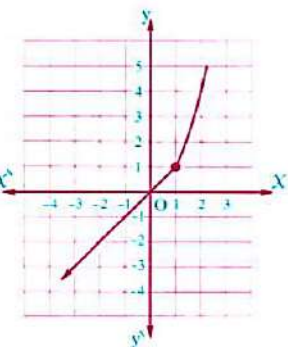
(a)



(b)



(c)



(d)

11)

In the opposite figure :

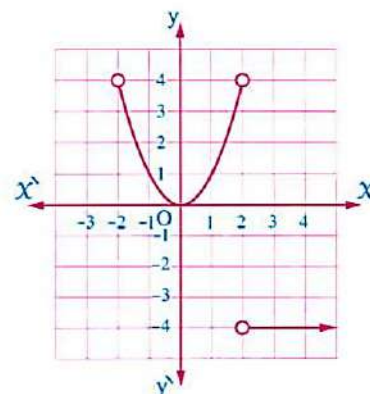
The curve of the function f defined by the rule $f(x) = \dots\dots\dots$

(a) $\begin{cases} x^2 & , \quad -2 < x < 2 \\ -4 & , \quad x < -2 \end{cases}$

(b) $\begin{cases} x^2 & , \quad -2 < x < 2 \\ -4 & , \quad x > 2 \end{cases}$

(c) $\begin{cases} x^2 & , \quad -2 \leq x \leq 2 \\ -4 & , \quad x < 2 \end{cases}$

(d) $\begin{cases} x^2 & , \quad -2 < x < 2 \\ -4 & , \quad x < 2 \end{cases}$



Lesson (5): Graphical transformations of functions of basic functions

To draw any given curve, you must get two elements:

- (i) The vertex point or the point of symmetry (P.O.S)
- (ii) The sign of the function

For example:

- (a) $F(x) = |x + 2| + 1$ $\therefore$ the vertex = $(-2, 1)$ +ve sign
- (b) $F(x) = (x - 1)^2$ $\therefore$ the vertex = $(1, 0)$ +ve sign
- (c) $F(x) = 3 - x^3$ $\therefore$ (P.O.S) = $(0, 3)$ -ve sign
- (d) $F(x) = |x| - 1$ $\therefore$ the vertex = $(0, 1)$ +ve sign
- (e) $F(x) = 2 - (x - 1)^2$ $\therefore$ the vertex = $(1, 2)$ -ve sign
- (f) $F(x) = \frac{1}{x-2}$ $\therefore$ (P.O.S) = $(2, 0)$ +ve sign

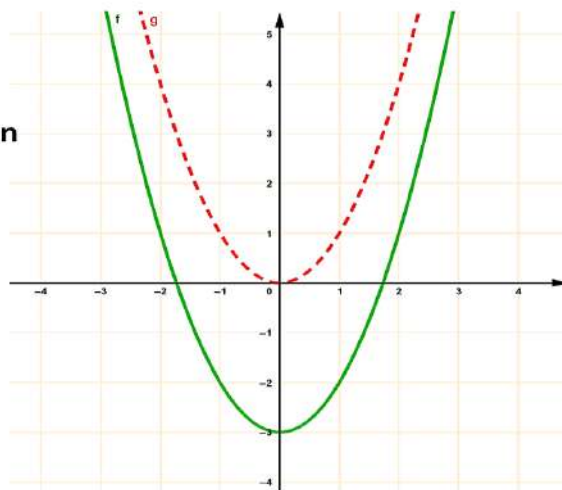


Use the curve of the function f where $g(x) = x^2$ to graph the curve of the function, then deduce its domain and its range and discuss its monotonicity and its type and write equation of axis of symmetry:

(1) $f(x) = x^2 - 3$

Sol.

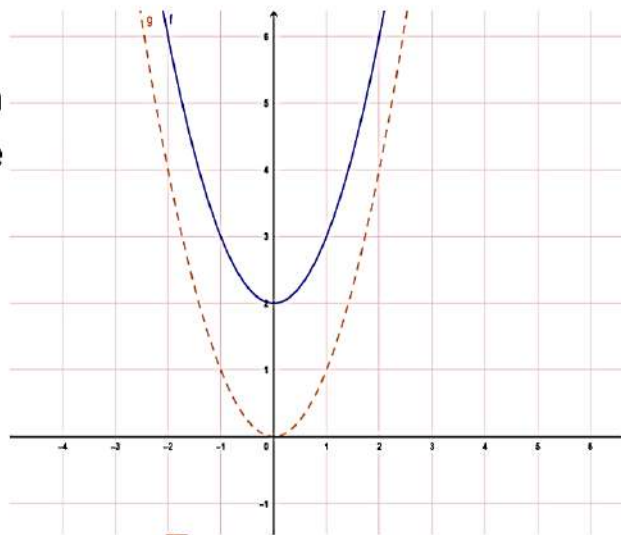
- Point of symmetry = $(0, -3)$
- Is the same curve of the function $g(x) = x^2$ by translation 3 units in the $-ve$ direction of y - axis
- Domain = R
- Range = $[-3, \infty[$
- Dec. = $] -\infty, 0[$
- Inc. = $] 0, \infty[$
- Even
- Equation : $x = 0$



(2) $f(x) = x^2 + 2$

Sol.

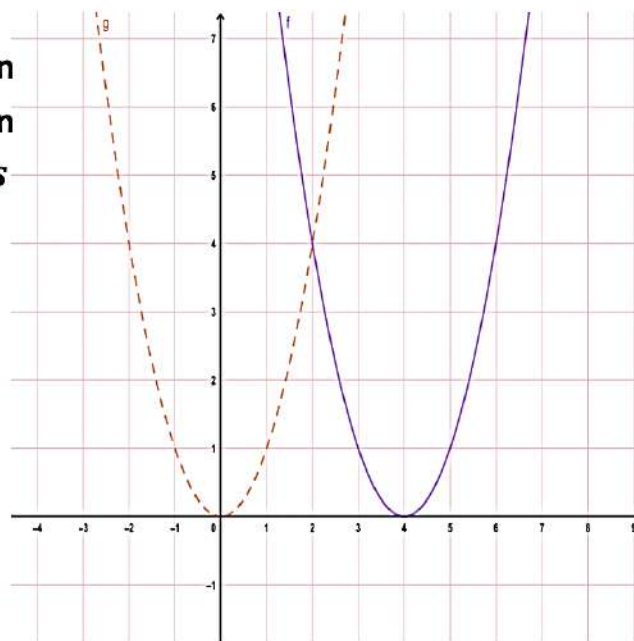
- Point of symmetry = $(0, 2)$
- Is the same curve of the function $g(x) = x^2$ translation 2 units in the *+ve* direction of *y - axis*
- Domain = R
- Range = $[2, \infty[$
- Dec. = $] -\infty, 0[$
- Inc. = $] 0, \infty[$
- Even , Equation : $x = 0$

**Second: The horizontal translation:**

(3) $f(x) = (x - 4)^2$

Sol.

- Point of symmetry = $(4, 0)$
- Is the same curve of the function $g(x) = x^2$ by horizontal translation 4 units in *+ve* direction of *x - axis*
- Domain = R
- Range = $[0, \infty[$
- Dec. = $] -\infty, 4[$
- Inc. = $] 4, \infty[$
- N.N
- Equation : $x = 4$

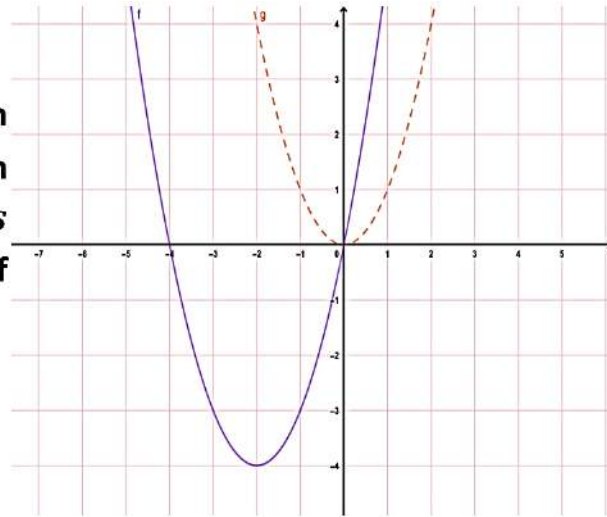


Third: The horizontal and vertical translation:

(4) $f(x) = (x + 2)^2 - 4$

Sol.

- Point of symmetry = $(-2, -4)$
- Is the same curve of the function $g(x) = x^2$ by horizontal translation 2 units in $-ve$ direction of x -axis and 4 units in the $-ve$ direction of y -axis
- Domain = R
- Range = $[-4, \infty[$
- Dec. = $]-\infty, 2[$
- Inc. = $]2, \infty[$
- N.N , Equation : $x = -2$



(5) $f(x) = x^2 - 4x + 5$

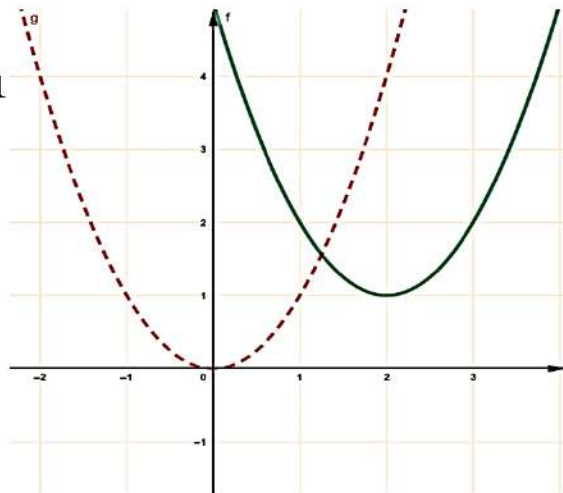
Sol.

Put the fn $f(x) = (x + a)^2 + b$ by completing $(x^2 - 4x)$ to be a perfect square

$$3^{\text{rd}} \text{ term} = \frac{(\text{Middle term})^2}{4 \times 1^{\text{st}}} = \frac{(-4x)^2}{4 \times x^2} = \frac{16x^2}{4x^2} = 4$$

$$\therefore f(x) = (x^2 - 4x + 4) + (-4 + 5) = (x - 2)^2 + 1$$

- Is the same curve of the function $g(x) = x^2$ by translation 2 units in the direction of $\overrightarrow{OX}$ and 1 unit in the direction of $\overrightarrow{OY}$
- Point of symmetry = $(2, 1)$
- Domain = R
- Range = $[1, \infty[$
- Dec. = $]-\infty, 2[$
- Inc. = $]2, \infty[$
- N.N



Use the curves of the standard functions to represent the curves of the functions:

(1) $F(x) = x - 2$ Domain = Range = Type Monotony One – to – one (yes or no)	(2) $F(x) = x + 1 - 2$ Domain = Range = Type Monotony One – to – one (yes or no)
(3) $F(x) = (x - 2)^2 - 1$ Domain = Range = Type One – to – one (yes or no) Increasing on Decreasing on	(4) $F(x) = \frac{2x^2 - 2}{x^2 - 1}$ Domain = Range = Type One – to – one (yes or no) Constant on
(5) $F(x) = \frac{x^2 + x}{x + 1}$ Domain = Range = Type Monotony One – to – one (yes or no)	(6) $F(x) = x^2 - 4x + 4$ Domain = Range = Type Monotony One – to – one (yes or no)

Use the curves of the standard functions to represent the curves of the functions:

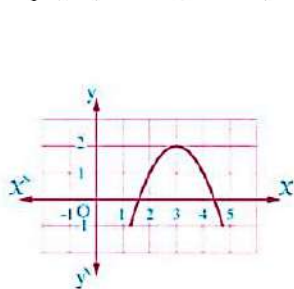
(1) $F(x) = -(x - 1)^3$	(2) $F(x) = 1 - x - 2$
Domain =	Domain =
Range =	Range =
Type	Type
Monotony	Monotony
One – to – one (yes or no)	One – to – one (yes or no)
(3) $F(x) = \frac{1}{2-x} + 1$	(4) $F(x) = -x^2 + 4x - 3$
Domain =	Domain =
Range =	Range =
Type	Type
One – to – one (yes or no)	Constant on
Increasing on Decreasing on	One – to – one (yes or no)
(5) $F(x) = 4 - x^2$	(6) $F(x) = 4 - x^2$
Domain =	Domain =
Range =	Range =
Type	Type
Monotony	Monotony
One – to – one (yes or no)	One – to – one (yes or no)

Exercises (5)

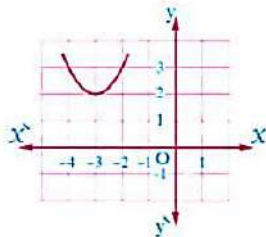
Choose the correct answer:

1)

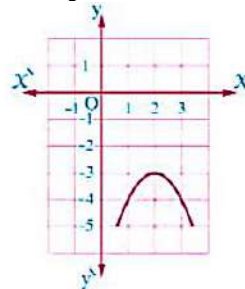
If $f(x) = -(x-3)^2 + 2$, then the graph that represents the function f is



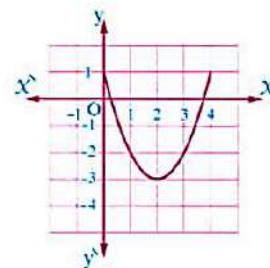
(a)



(b)



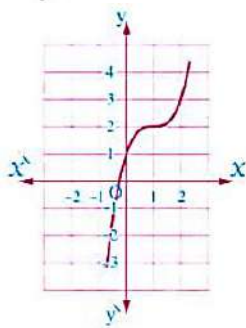
(c)



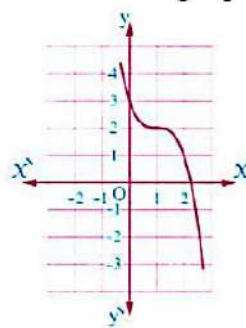
(d)

2)

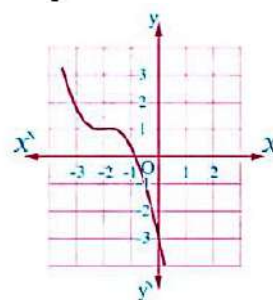
If $f(x) = 2 - (x-1)^3$, then the graph that represents the function f is



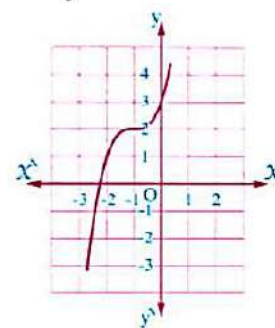
(a)



(b)



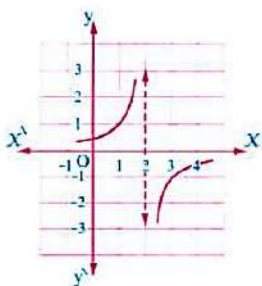
(c)



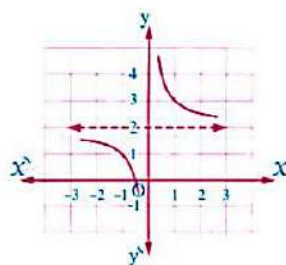
(d)

3)

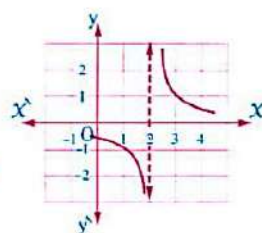
If $f(x) = \frac{1}{x-2}$, then the graph that represents the function f is



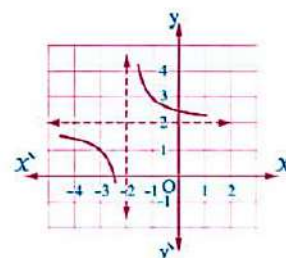
(a)



(b)



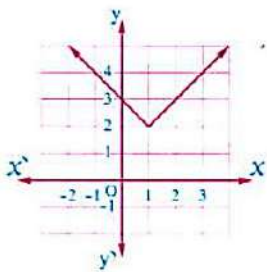
(c)



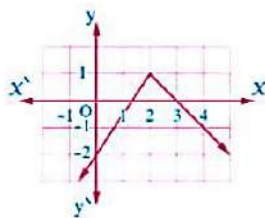
(d)

4)

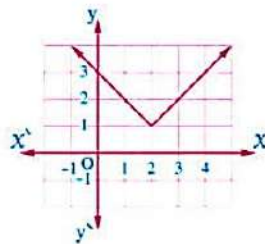
If $f : f(x) = 1 - |x - 2|$ then the figure which represents the function f is



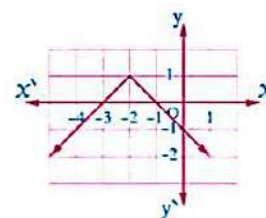
(a)



(b)



(c)



(d)

5)

If the curve of the function $g : g(x) = x^2$ is translated two units in the positive directions of the two axes then the function represents this translation is f :

(a) $f(x) = (x + 2)^2 + 2$

(b) $f(x) = (x + 2)^2 - 2$

(c) $f(x) = (x - 2)^2 - 2$

(d) $f(x) = (x - 2)^2 + 2$

6)

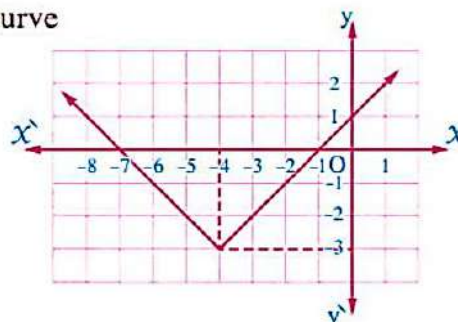
Which of the following function rules represents the curve in the given figure ?

(a) $f(x) = |x - 4| - 3$

(b) $f(x) = |x - 4| + 3$

(c) $f(x) = |x + 4| - 3$

(d) $f(x) = |x + 4| + 3$



7)

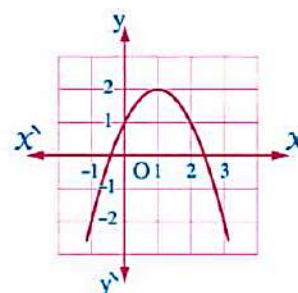
Which of the following function rules is represented in the given figure ?

(a) $f(x) = (x - 1)^2 + 2$

(b) $f(x) = 1 - (x - 2)^2$

(c) $f(x) = 2 - (x - 1)^2$

(d) $f(x) = (x + 1)^2 - 2$



8)

 The point of the vertex of the curve of the function $f : f(x) = (2 - x)^2 + 3$ is

(a) (2, 3)

(b) (2, -3)

(c) (-2, 3)

(d) (-2, -3)

9)	The symmetric point of the function $f : f(x) = 3 - (x + 2)^2$ is	(a) (3 , 2)	(b) (2 , 3)	(c) (-2 , 3)	(d) (-2 , -3)
10)	 The point of symmetry of the curve of the function $f : f(x) = \frac{1}{x-3} + 4$ is	(a) (3 , -4)	(b) (-3 , -4)	(c) (3 , 4)	(d) (-3 , 4)
11)	The symmetric point of the function $f : f(x) = \frac{x+1}{x}$ is	(a) (1 , 0)	(b) (0 , 1)	(c) (0 , 0)	(d) (1 , -1)
12)	 If f is a function where $f(x) = \frac{1}{x}$, then the symmetric point of the function $g : g(x) = f(x + 1)$ is	(a) (1 , 0)	(b) (0 , 1)	(c) (-1 , 0)	(d) (-1 , 1)
13)	The vertex of the curve of the function $f : f(x) = x + 3 - 2$ is	(a) (3 , 2)	(b) (-3 , -2)	(c) (-3 , 2)	(d) (3 , -2)
14)	The curve of the function $f : f(x) = x - 2 $ is symmetric about the straight line	(a) $x = 2$	(b) $x = -2$	(c) $y = 2$	(d) $y = -2$
15)	The range of the function $f : f(x) = \frac{1}{ x }$ is	(a) $\mathbb{R} - \{0\}$	(b) $]0 , \infty[$	(c) $[0 , \infty[$	(d) $\{0\}$
16)	The range of the function $f : f(x) = \frac{2x^2 - 2x}{x - 1}$ equals	(a) $\mathbb{R} - \{1\}$	(b) $\mathbb{R} - \{2\}$	(c) $\mathbb{R} - \{1 , 2\}$	(d) $\mathbb{R} - \{0 , 1\}$
17)	The range of the function $f : f(x) = 2 - 3 - 2x $ is	(a) $] - \infty , 2]$	(b) $[-2 , \infty[$	(c) $] \frac{3}{2} , \infty[$	(d) $] - \infty , -2]$

18)	If $y = f(x)$ is a real function , then its image by translation 3 units vertically upwards is $g(x) = \dots\dots\dots$ (a) $f(x - 3)$ (b) $f(x + 3)$ (c) $f(x) + 3$ (d) $(x) - 3$
19)	 The curve of the function $g : g(x) = x^2 + 4$ is the same curve of the function $f : f(x) = x^2$ by a translation of magnitude 4 units in the direction of $\dots\dots\dots$ (a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy}$
20)	The curve of the function g where $g(x) = x - 2$ is the same as the curve of the function $f : f(x) = x$ by translation two units in direction of $\dots\dots\dots$ (a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy}$
21)	 The curve of the function $g : g(x) = x + 3$ is the same curve of the function $f : f(x) = x$ by a translation of magnitude 3 units in the direction of $\dots\dots\dots$ (a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy}$
22)	If $y = f(x)$ is a real function , then its image by translation 4 units to the left is $g(x) = \dots\dots\dots$ (a) $f(x - 4)$ (b) $f(x + 4)$ (c) $f(x) + 4$ (d) $f(x) - 4$

Lesson (1): Introduction to limits of functions

Specified, unspecified, and undefined quantities

1 Specified quantity

It is the quantity which has determined result , for example $\frac{2}{3}$ is a specified quantity .

2 Unspecified quantity

It is the quantity which has no determined result , for example $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \times \infty$, $\infty - \infty$, $(\text{zero})^{\text{zero}}$, $(\infty)^{\text{zero}}$ and $(1)^{\infty}$

3 Undefined quantity

It is the quantity which is meaningless , for example $\frac{4}{0}$ is undefined quantity .

Properties of ∞ , $-\infty$

$$(1) \infty \pm a = \infty , -\infty \pm a = -\infty$$

$$(2) \infty \times a = \begin{cases} \infty & , a > 0 \\ -\infty & , a < 0 \\ \text{unspecified} & , a = 0 \end{cases}$$

The concept of the limit of a function at a point

Illustrated example

If we want to find the value of the function $f : f(x) = \frac{x^4 - 1}{x - 1}$ at $x = 1$, we find its value = $\frac{0}{0}$ which is unspecified value .

That mean we can not determine the value of the function at $x = 1$, so we go to study the approaching of $f(x)$ to a specified quantity , when x approach to the number 1 , that by one of the following two methods :

1 Evaluation of the limit numerically:

x approaches to 1 from left ----->						<----- x approaches to 1 from right					
x	0.5	0.6	0.7	0.8	0.9		1.1	1.2	1.3	1.4	1.5
f(x)	1.5	1.6	1.7	1.8	1.9		2.1	2.2	2.3	2.4	2.5
f(x) approaches to 1 from left ----->						<----- f(x) approaches to 1 from right					

We find that:

When x approaches to the number 1 from the right side " $x \rightarrow 1^+$ " and is read as : (x tends to one from the right) , then f(x) approaches to the number 2 and the number 2 is called the right limit of the function and it is written mathematically as : $\lim_{x \rightarrow 1^+} f(x) = 2$ or $f(1^+) = 2$ and is read as the limit of the function when ($x \rightarrow 1^+$) equals 2

When x approaches to the number 1 from the left side " $x \rightarrow 1^-$ " and is read as : (x tends to one from the left) , then f(x) approaches to the number 2 and the number 2 is called the right limit of the function and it is written mathematically as : $\lim_{x \rightarrow 1^-} f(x) = 2$ or $f(1^-) = 2$ and is read as the limit of the function when ($x \rightarrow 1^-$) equals 2

Definition

If the value of the function f approaches to a unique value L when x approaches to a from the two sides right and left , then the limit of f(x) equals L and is written symbolically :

$$\lim_{x \rightarrow a} f(x) = L$$

i.e. If $f(a^+) = f(a^-) = L$, then $\lim_{x \rightarrow a} f(x) = L$

In the previous example : $f(1^+) = f(1^-) = 2$, then $\lim_{x \rightarrow 1} f(x) = 2$

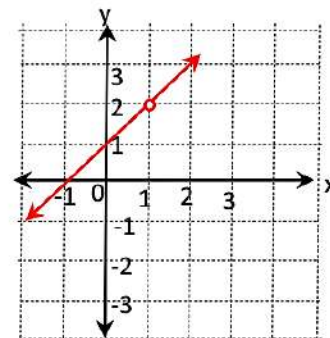
② Evaluation of the limit graphically:

$\therefore f(x) = \frac{x^2 - 1}{x - 1}$ is undefined at $x = 1$

$$\therefore f(x) = \frac{(x - 1)(x + 1)}{(x - 1)} = (x + 1), x \neq 1$$

It is represented by a straight line with an open dot at the point whose x-coordinate = 1 as in the opposite figure, and from the figure we notice that when x tends to 1 (from the right and the left), then $f(x)$ tends to 2

$$\therefore \lim_{x \rightarrow 1} f(x) = 2$$

**Remarks:**

[1] At finding $\lim_{x \rightarrow a} f(x)$, it is not necessary that the function be defined at $x = a$, it should be defined only in an interval on the left of a and another interval on the right of a .

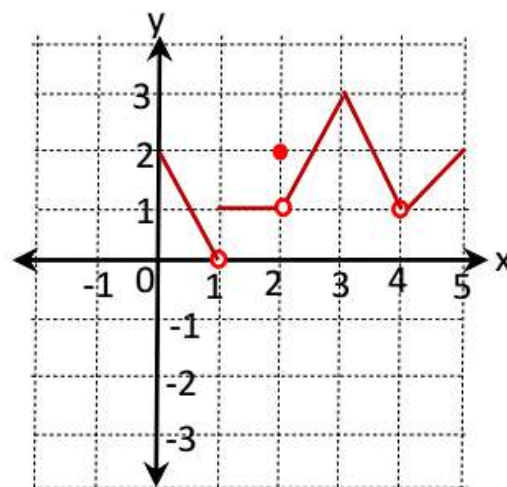
[2] If $f(a^+) \neq f(a^-)$, then $\lim_{x \rightarrow a} f(x)$ does not exist.

(1) In the opposite figure, we find that :

First : at $x = 0$: $\lim_{x \rightarrow 0^+} f(x) = 1$

But $\lim_{x \rightarrow 0^-} f(x)$ and $\lim_{x \rightarrow 0} f(x)$ don't exist

(Because the function is undefined at the left of $x = 0$)



Second : at $x = 1$: $\lim_{x \rightarrow 1^-} f(x) = 0$, $\lim_{x \rightarrow 1^+} f(x) = 1$

$\therefore \lim_{x \rightarrow 1} f(x)$ does not exist (because the right limit $\neq$ the left limit)

Notice that : although f is defined at $x = 1$, $f(1) = 1$, the limit does not exist.

Third : at $x = 2$: $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} f(x) = 1$

Notice that : it is not necessary that the value of the function equals the value of the limit where $f(2) = 2$

Fourth : at $x = 3$: $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3} f(x) = f(3) = 3$

Fifth : at $x = 4$: $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4} f(x) = 1$

Notice that : $f(4)$ is undefined i.e. Although the function is undefined , the limit exists

Sixth : at $x = 5$: $\lim_{x \rightarrow 5^-} f(x) = 2$, $\lim_{x \rightarrow 5^+} f(x)$ and $\lim_{x \rightarrow 5} f(x)$ don't exist

(Because the function is undefined on the right of $x = 5$)

Examples

[1] Estimate the existence of a limit of each of the following functions when $x = 2$

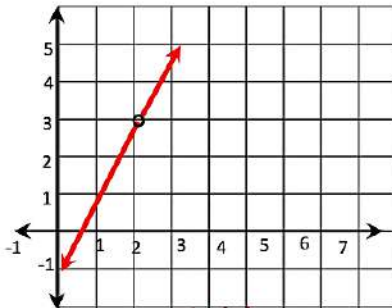


Fig (1)

$$F(2^+) =$$

$$F(2^-) =$$

$$\lim_{x \rightarrow 2} F(x) =$$

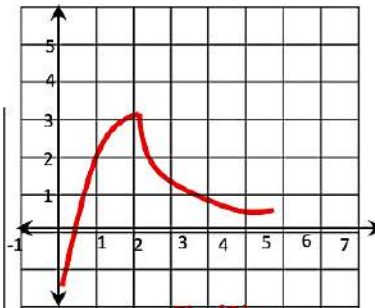


Fig (2)

$$F(2^+) =$$

$$F(2^-) =$$

$$\lim_{x \rightarrow 2} F(x) =$$

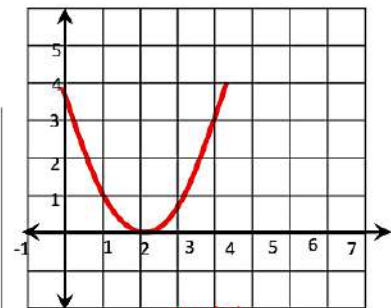


Fig (3)

$$F(2^+) =$$

$$F(2^-) =$$

$$\lim_{x \rightarrow 2} F(x) =$$

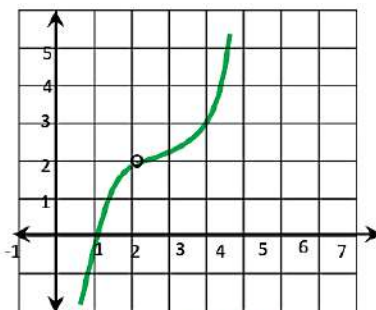


Fig (4)

$$F(2^+) =$$

$$F(2^-) =$$

$$\lim_{x \rightarrow 2} F(x) =$$

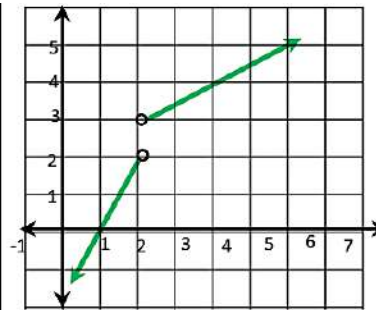


Fig (5)

$$F(2^+) =$$

$$F(2^-) =$$

$$\lim_{x \rightarrow 2} F(x) =$$

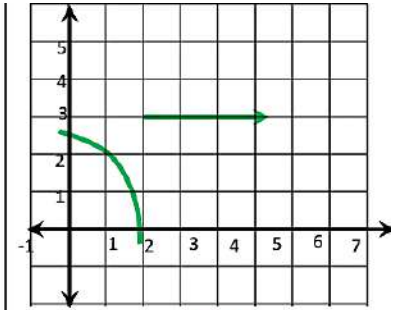


Fig (6)

$$F(2^+) =$$

$$F(2^-) =$$

$$\lim_{x \rightarrow 2} F(x) =$$

[2] From the opposite figures:

$$F(0^+) =$$

$$F(0^-) =$$

$$\lim_{x \rightarrow 0} F(x) =$$

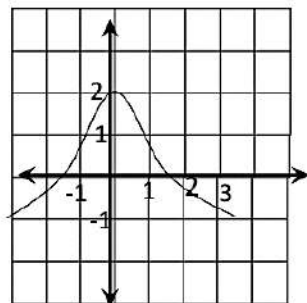


Fig (1)

$$F(1^+) =$$

$$F(1^-) =$$

$$\lim_{x \rightarrow 1} F(x) =$$

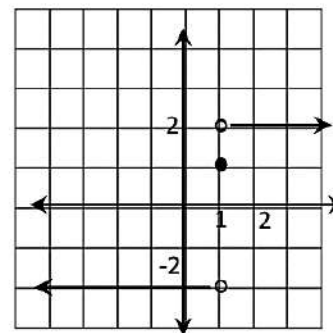


Fig (2)

$$F(2^+) =$$

$$F(2^-) =$$

$$\lim_{x \rightarrow 2} F(x) =$$

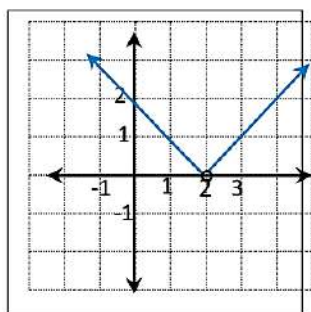


Fig (3)

$$F(-1^+) =$$

$$F(-1^-) =$$

$$\lim_{x \rightarrow -1} F(x) =$$

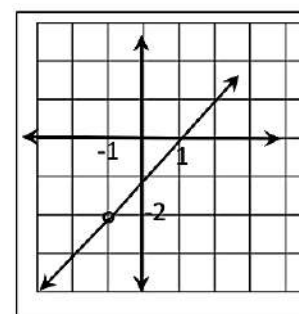


Fig (4)

$$F(0^+) =$$

$$F(0^-) =$$

$$\lim_{x \rightarrow 0} F(x) =$$

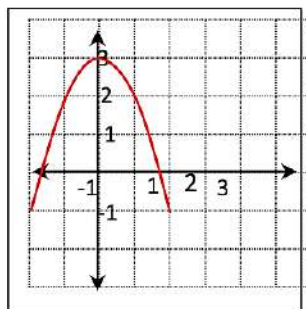


Fig (5)

$$F(-1^+) =$$

$$F(-1^-) =$$

$$\lim_{x \rightarrow -1} F(x) =$$

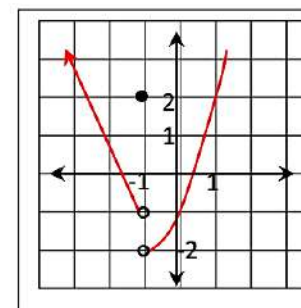


Fig (6)

$$F(1^+) =$$

$$F(1^-) =$$

$$\lim_{x \rightarrow 1} F(x) =$$

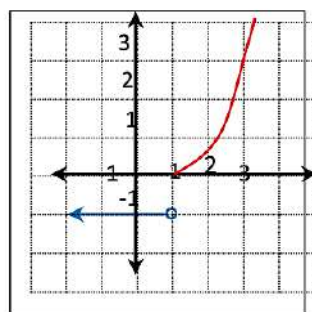


Fig (7)

$$F(-1^+) =$$

$$F(-1^-) =$$

$$\lim_{x \rightarrow -1} F(x) =$$

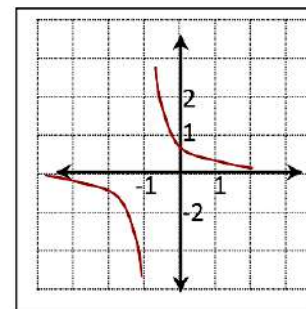
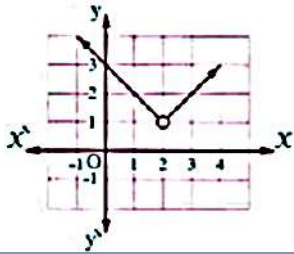
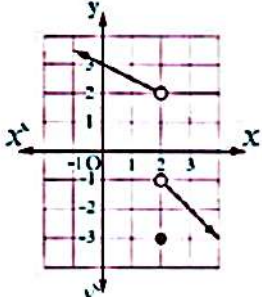
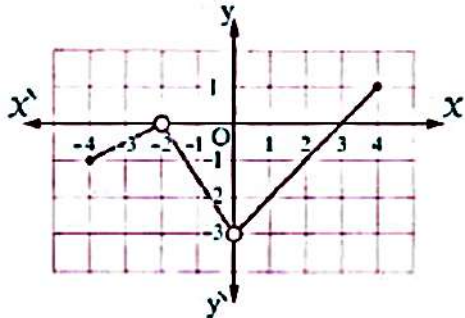


Fig (8)

Exercises (1)

Choose the correct answer:

1)	All the following are unspecified quantities except	
	(a) zero $\div$ zero (b) $\infty - \infty$ (c) $\infty + \infty$ (d) $\infty \div \infty$	
2)	In the opposite figure : $\lim_{x \rightarrow 2} f(x) = \dots\dots\dots$	
	(a) 1 (b) -1 (c) does not exist. (d) 2	
3)	In the opposite figure : $\lim_{x \rightarrow 2} f(x) = \dots\dots\dots$	
	(a) -3 (b) 2 (c) -1 (d) does not exist.	
4)	 From the opposite figure : First : $\lim_{x \rightarrow -2} f(x) = \dots\dots\dots$	
	(a) zero (b) -3 (c) -2 (d) does not exist.	
	Second : $\lim_{x \rightarrow 0} f(x) = \dots\dots\dots$	
	(a) zero (b) -2 (c) -3 (d) does not exist.	
	Third : $\lim_{x \rightarrow -4} f(x) = \dots\dots\dots$	
	(a) zero (b) -4 (c) -1 (d) does not exist.	
	Fourth : $\lim_{x \rightarrow 4} f(x) = \dots\dots\dots$	
	(a) zero (b) 4 (c) 1 (d) does not exist.	

5)

Using the opposite figure :

First : $\lim_{x \rightarrow 1^+} f(x) = \dots\dots\dots$

- (a) zero (b) -2
(c) 1 (d) does not exist.

Second : $\lim_{x \rightarrow 1^-} f(x) = \dots\dots\dots$

- (a) zero (b) -2 (c) 1 (d) does not exist.

Third : $\lim_{x \rightarrow 1} f(x) = \dots\dots\dots$

- (a) zero (b) -2 (c) 1 (d) does not exist.

Fourth : $\lim_{x \rightarrow -1^+} f(x) = \dots\dots\dots$

- (a) zero (b) -1 (c) -2 (d) does not exist.

Fifth : $\lim_{x \rightarrow -1^-} f(x) = \dots\dots\dots$

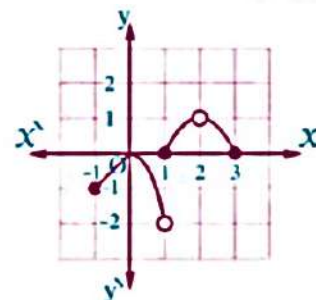
- (a) zero (b) -1 (c) -2 (d) does not exist.

Sixth : $\lim_{x \rightarrow 2} f(x) = \dots\dots\dots$

- (a) zero (b) 1 (c) 2 (d) does not exist.

Seventh : $\lim_{x \rightarrow 0} f(x) = \dots\dots\dots$

- (a) zero (b) -1 (c) -2 (d) does not exist.



6)

Using the opposite figure :

First : $\lim_{x \rightarrow -1^+} f(x) = \dots\dots\dots$

- (a) zero (b) -1
(c) 4 (d) does not exist.

Second : $\lim_{x \rightarrow -1^-} f(x) = \dots\dots\dots$

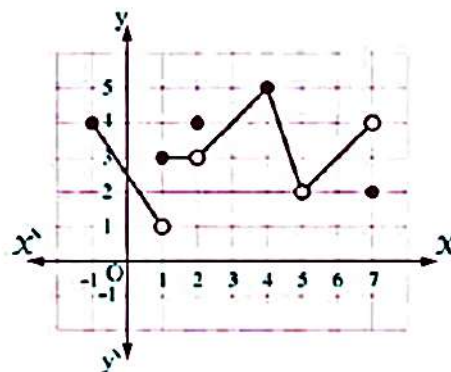
- (a) zero (b) -1 (c) 4 (d) does not exist.

Third : $\lim_{x \rightarrow -1} f(x) = \dots\dots\dots$

- (a) zero (b) -1 (c) 4 (d) does not exist.

Fourth : $f(2) = \dots\dots\dots$

- (a) zero (b) 3 (c) 4 (d) undefined.



Lesson (2): Finding the limit of a function algebraically

Direct substitution:

Theorem ①: If $f(x)$ is a polynomial, $a \in \mathbb{R}$, then $\lim_{x \rightarrow a} f(x) = f(a)$
i.e (Direct substitution)

Examples

[1] Find each of the following limits:

(a) $\lim_{x \rightarrow 2} (x^2 - 3x + 5)$

(b) $\lim_{x \rightarrow 3} (-5)$

(c) $\lim_{x \rightarrow 3} (2x - 5)$

(d) $\lim_{x \rightarrow -2} (3x^2 + x - 4)$

(e) $\lim_{x \rightarrow 2} \frac{3}{x-2}$

(f) $\lim_{x \rightarrow -5} \frac{x+5}{3x}$

Theorem ②: If f, g are two real functions in x , $\lim_{x \rightarrow a} f(x) = n$, $\lim_{x \rightarrow a} g(x) = m$ where :
 $L, m \in \mathbb{R}$, then :

(1) $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = n \pm m$

(2) $\lim_{x \rightarrow a} [f(x) \times g(x)] = \lim_{x \rightarrow a} f(x) \times \lim_{x \rightarrow a} g(x) = n \times m$

(3) $\lim_{x \rightarrow a} k f(x) = k \lim_{x \rightarrow a} f(x) = k n$ where k is constant

(4) $\lim_{x \rightarrow a} [f(x) \div g(x)] = \lim_{x \rightarrow a} f(x) \div \lim_{x \rightarrow a} g(x) = n \div m, m \neq 0$

(5) $\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n = L^n, n \in \mathbb{Z}^+$

Using factorization:

[2] Find each of the following limits:

$$(1) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

$$(2) \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{2x - 4}$$

$$(3) \lim_{x \rightarrow -3} \frac{x^2 + 8x + 15}{x^2 - 4x - 21}$$

$$(4) \lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$$

$$(5) \lim_{x \rightarrow \sqrt{3}} \frac{x^2 - 3}{\sqrt{3}x - 3}$$

$$(6) \lim_{x \rightarrow -2} \frac{x^4 - 3x^2 - 4}{2x^2 - 8}$$

$$(7) \lim_{x \rightarrow -2} \frac{x^2 - 2x - 8}{x^3 + 8}$$

$$(8) \lim_{x \rightarrow -0} \frac{(x-1)^2 - 1}{4x}$$

$$(9) \lim_{x \rightarrow -0} \frac{(3x-2)^2 - 4}{5x}$$

$$(10) \lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x^2 + 5x - 3}$$

$$(11) \lim_{x \rightarrow \frac{3}{2}} \frac{8x^3 - 27}{2x^2 + 3x - 9}$$

$$(12) \lim_{x \rightarrow 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$$

Finding limits by using long division method

[3] Find each of the following limits:

(1) $\lim_{x \rightarrow 4} \frac{x^3 - 15x - 4}{x - 4}$

(2) $\lim_{x \rightarrow 2} \frac{3x^3 - 5x^2 - 3x + 2}{x^3 - 8}$

Sol.

(1) $\lim_{x \rightarrow 4} \frac{x^3 - 15x - 4}{x - 4} = \frac{0}{0} = \text{unspecified value}$

$\therefore \lim_{x \rightarrow 4} \frac{(x-4)(x^2 + 4x + 1)}{(x-4)} = \lim_{x \rightarrow 4} (x^2 + 4x + 1) = 33$

1	0	-15	-4	4
0	4	16	4	
1	4	1	0	

(3) $\lim_{x \rightarrow 1} \frac{x^3 + x^2 - 10x + 8}{x^3 + x^2 - 17x + 15}$

(4) $\lim_{x \rightarrow 1} \frac{5x^{-3} + 2x^{-1} - 7}{3x^{-2} + x^{-3} - 4}$

Finding limits by using the conjugate

[3] Find each of the following limits:

$$(1) \lim_{x \rightarrow 4} \frac{\sqrt{x-3}-1}{x-4}$$

$$(2) \lim_{x \rightarrow 5} \frac{\sqrt{x-1}-2}{x-5}$$

$$(3) \lim_{x \rightarrow -1} \frac{x+1}{\sqrt{x+5}-2}$$

$$(4) \lim_{x \rightarrow 5} \frac{x^2-5x}{\sqrt{x+4}-3}$$

$$(5) \lim_{x \rightarrow 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$$

$$(6) \lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{\sqrt{x-2}-1}$$

Exercises (2)

Choose the correct answer:

1)	$\lim_{x \rightarrow \frac{1}{2}} (10) = \dots\dots\dots$ (a) 5 (b) 20 (c) 10 (d) $10\frac{1}{2}$
2)	$\lim_{x \rightarrow 2} (3 a^2) = \dots\dots\dots$ (a) 3 (b) 6 (c) $3 a^2$ (d) 12
3)	$\lim_{x \rightarrow \frac{\pi}{2}} (2 x - \sin x) = \dots\dots\dots$ (a) π (b) $\pi + 1$ (c) $\pi - 1$ (d) $2 \pi - 1$
4)	$\lim_{x \rightarrow 3} \frac{2 x - 6}{7 x - 21} = \dots\dots\dots$ (a) $\frac{2}{3}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) 3
5)	$\lim_{x \rightarrow 0} \frac{x^2 - x}{x} = \dots\dots\dots$ (a) zero. (b) -1 (c) does not exist. (d) 1
6)	$\lim_{x \rightarrow 3} \frac{x^2 - 7 x + 12}{x - 3} = \dots\dots\dots$ (a) 1 (b) -1 (c) 7 (d) -2
7)	$\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 + x - 12} = \dots\dots\dots$ (a) $\frac{5}{7}$ (b) $\frac{1}{7}$ (c) -1 (d) -5

8)	$\lim_{x \rightarrow 0} \frac{3x + 2x^{-1}}{x + 4x^{-1}} = \dots\dots\dots$ (a) 2 (b) 4 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
9)	$\lim_{x \rightarrow \sqrt{5}} \frac{x^4 - x^2 - 20}{x - \sqrt{5}} = \dots\dots\dots$ (a) 9 (b) $2\sqrt{5}$ (c) $9\sqrt{5}$ (d) $18\sqrt{5}$
10)	$\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} = \dots\dots\dots$ (a) zero (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) has no existence.
11)	$\lim_{x \rightarrow 9} \frac{\sqrt{2} - \sqrt{x-7}}{x-9} = \dots\dots\dots$ (a) $2\sqrt{2}$ (b) $\frac{\sqrt{2}}{4}$ (c) $-\frac{\sqrt{2}}{4}$ (d) $-2\sqrt{2}$
12)	$\lim_{x \rightarrow 2} \frac{(x-3)^2 - 1}{\sqrt{x+2} - 2} = \dots\dots\dots$ (a) -6 (b) -8 (c) -2 (d) does not exist.
13)	$\lim_{x \rightarrow 1} \frac{\sqrt{2x-1} - 1}{\sqrt{3x+1} - 2} = \dots\dots\dots$ (a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{4}{3}$
14)	$\lim_{x \rightarrow 2} \frac{x^3 - 7x + 6}{3x^2 - 8x + 4} = \dots\dots\dots$ (a) $\frac{5}{4}$ (b) $\frac{3}{2}$ (c) $\frac{4}{5}$ (d) $\frac{1}{3}$

15)	$\lim_{x \rightarrow 0} \frac{7+2x}{\cos x} = \dots\dots\dots$ (a) 7 (b) 8 (c) 9 (d) 1
16)	$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x}{x} = \dots\dots\dots$ (a) 0 (b) 1 (c) $\frac{4}{\pi}$ (d) does not exist.
17)	If $\lim_{x \rightarrow m} \frac{2x^2 - x - 3}{4x^2 - 9} = \frac{5}{12}$, then m = $\dots\dots\dots$ (a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$
18)	If $\lim_{x \rightarrow 2} \frac{x^2 - 4a}{x - 2}$ exists, then a = $\dots\dots\dots$ (a) -1 (b) 1 (c) 2 (d) 4

Lesson (1): The sine rule

In any triangle, the lengths of the sides are proportional to the sines of the opposite angles i.e $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

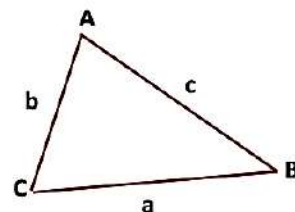
Proof: Area of triangle = $\frac{1}{2} b c \sin A = \frac{1}{2} a c \sin B = \frac{1}{2} a b \sin C$ (Multiplying by 2)

$$\therefore b c \sin A = a c \sin B = a b \sin C \text{ (dividing by } a b c \text{)}$$

$$\therefore \frac{b c \sin A}{a b c} = \frac{a c \sin B}{a b c} = \frac{a b \sin C}{a b c}$$

$$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Using the sine rule to find the length of a side of a triangle

[1] Find the length of the longest side in the triangle ABC in which $m(\angle A) = 54^\circ 33'$, $m(\angle B) = 49^\circ 22'$, $a = 124.5$ cm.

Sol.

$\therefore$ The longest side is opposite to the greatest angle

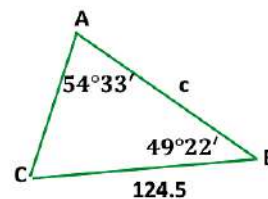
$$m(\angle C) = 180^\circ - [m(\angle A) + m(\angle B)] =$$

$$180^\circ - [54^\circ 33' + 49^\circ 22'] = 76^\circ 5'$$

$\therefore$ The longest side is c

$$\therefore \frac{a}{\sin A} = \frac{c}{\sin C} \quad \therefore \frac{124.5}{\sin 54^\circ 33'} = \frac{c}{\sin 76^\circ 5'}$$

$$\therefore c = \frac{124.5 \sin 76^\circ 5'}{\sin 54^\circ 33'} = 148.4 \text{ cm}$$



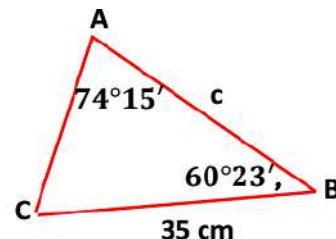
[2] In the triangle ABC in which $m(\angle A) = 74^\circ 15'$, $m(\angle B) = 60^\circ 23'$, $a = 35$ cm. Find a, b.

Sol.

$$m(\angle C) = 180^\circ - [74^\circ 15' + 60^\circ 23'] = 45^\circ 22'$$

$$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \therefore \frac{35}{\sin 74^\circ 15'} = \frac{b}{\sin 60^\circ 23'} = \frac{c}{\sin 45^\circ 22'}$$

$$\therefore b = 31.6 \text{ cm}, c = 25.9 \text{ cm.}$$



[3] In the triangle ABC in which $m(\angle A) = 110^\circ$, $m(\angle B) = 35^\circ$, $a = 436.2$ cm.
Find a, b, and the area of ΔABC .

Sol.

[4] ABCD is a parallelogram, $AB = 8$ cm, $m(\angle BAC) = 55^\circ$, $m(\angle ABD) = 20^\circ$. Find:
(i) The length of $\overline{BD}$, $\overline{AC}$
(ii) S. area of the parallelogram ABCD.

Sol.

Geometric Application of the Sine Rule:

In any triangle ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$

Where r is the length of the radius of the circumcircle of ΔABC

[5] In ΔABC , if $a = 4$ cm, $m(\angle A) = 40^\circ$. Find the surface area of the circumcircle of ΔABC .

Sol.

$$\therefore \frac{a}{\sin A} = 2r \quad \therefore 2r = \frac{4}{\sin 40^\circ} \quad \therefore r = 3.1 \text{ cm}$$

$$\therefore \text{S. area of the circumcircle of } \Delta ABC = \pi r^2 = \pi (3.1)^2 = 30.2 \text{ cm}$$

[6] In ΔABC , if $m(\angle A) = 32^\circ$, $m(\angle B) = 126^\circ$ and its perimeter = 79 cm. Find:

i) The diameter of the circle passing through its vertices.

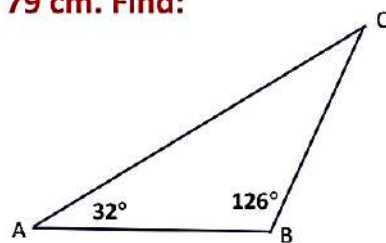
ii) The length of the smallest side.

Sol.

$$m(\angle C) = 180^\circ - (126^\circ + 32^\circ) = 22^\circ$$

$$\therefore \frac{\text{Perimeter of } \Delta ABC}{\sin A + \sin B + \sin C} = 2r \quad \therefore 2r = \frac{79}{1.7135} = 46.11 \text{ cm.}$$

$$\therefore \text{The diameter of the circle} = 2r = 46.11 \text{ cm.}$$



[7] Prove that the area of $\Delta ABC = \frac{abc}{4r}$ where r is the length of the radius passing through

its vertices, and if its area = $40\sqrt{3}$ cm², $b = 16$ cm, $c = 10$ cm, $r = \frac{14\sqrt{3}}{3}$ cm.

Find a and $m(\angle A)$.

Sol.

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} ab \sin C, \quad \therefore \frac{c}{\sin 22^\circ} = 2r \quad \therefore \sin C = \frac{c}{2r}$$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} ab \times \frac{c}{2r} = \frac{abc}{4r}$$

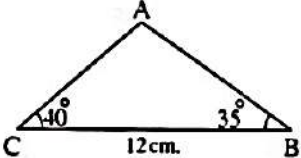
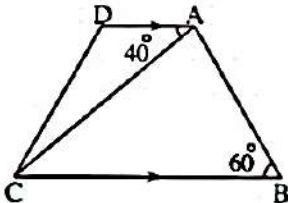
$$\therefore 40\sqrt{3} = \frac{a \times 16 \times 10}{4 \times \frac{14\sqrt{3}}{3}} \quad \therefore a = \frac{4 \times 14\sqrt{3} \times 40\sqrt{3}}{3 \times 16 \times 10} = 14 \text{ cm}$$

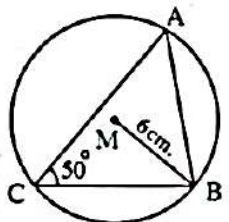
$$\therefore \frac{a}{\sin A} = 2r \quad \therefore \frac{14}{\sin A} = 2 \times \frac{14\sqrt{3}}{3} \quad \therefore 28\sqrt{3} \sin A = 42$$

$$\therefore \sin A = \frac{\sqrt{3}}{2} \quad \therefore m(\angle A) = 60^\circ \text{ or } 120^\circ$$

Exercises (1)

Choose the correct answer:

1)	In any triangle XYZ , XY : YZ =	
	(a) $\sin X : \sin Y$ (b) $\sin Y : \sin Z$ (c) $\sin Z : \sin X$ (d) $\sin Z : \sin Y$	
2)	In ΔABC , if $m(\angle A) = 30^\circ$, $C = 15\sqrt{3}$ cm. , $m(\angle C) = 60^\circ$, then $a = \dots\dots\dots$ cm.	
	(a) 30 (b) 45 (c) 15 (d) 60	
3)	DEF is a triangle in which $m(\angle D) = 80^\circ$ and $m(\angle E) = 60^\circ$, if $f = 12$ cm. , then $d = \dots\dots\dots$ cm.	
	(a) $\frac{12 \sin 80^\circ}{\sin 40^\circ}$ (b) $\frac{12 \sin 80^\circ}{\sin 60^\circ}$ (c) $\frac{12 \sin 40^\circ}{\sin 80^\circ}$ (d) $\frac{12 \cos 80^\circ}{\cos 40^\circ}$	
4)	In ΔABC , if $a = 4$ cm. , $b = 7$ cm. , $m(\angle C) = 120^\circ$, then the area of the triangle = $\dots\dots\dots$ cm^2 .	
	(a) $7\sqrt{3}$ (b) $14\sqrt{3}$ (c) 7 (d) 14	
5)	 XYZ is an equilateral triangle , the length of its side is $10\sqrt{3}$ cm. , then the length of the diameter of its circumcircle is $\dots\dots\dots$ cm.	
	(a) 5 (b) 10 (c) 15 (d) 20	
6)	In the opposite figure : The length of $\overline{AB} \approx \dots\dots\dots$ cm.	
	(a) 6 (b) 7 (c) 8 (d) 9	
7)	In the opposite figure : $\overline{AD} \parallel \overline{BC}$, $AB = 4$ cm. , $m(\angle DAC) = 40^\circ$, $m(\angle B) = 60^\circ$, then the length of $\overline{AC} \approx \dots\dots\dots$ cm.	
	(a) 5 (b) 3 (c) 2 (d) 4	

8)	In the opposite figure : M is the centre of the circle BM = 6 cm. , then AB = cm. (a) $6 \sin 50^\circ$ (b) $12 \sin 50^\circ$ (c) $6 \cos 50^\circ$ (d) $12 \cos 50^\circ$	
9)	A circle with diameter of length 20 cm. , passes through the vertices of $\triangle ABC$ which is an acute-angled triangle in which $BC = 10$ cm. , then $m(\angle A) = \dots\dots\dots^\circ$ (a) 30 (b) 60 (c) 45 (d) 150	
10)	In triangle ABC , $m(\angle A) = 45^\circ$, the length of the radius of its circumcircle = 6 cm. , then $a = \dots\dots\dots$ cm. (a) 13 (b) $6\sqrt{2}$ (c) 12 (d) $\sqrt{2}$	
11)	If the perimeter of triangle ABC equals 15 cm. , $m(\angle A) = 53^\circ$, $m(\angle B) = 47^\circ$, then the length of $\overline{AB} = \dots\dots\dots$ cm. (a) 6 (b) 7 (c) 5 (d) 8	
12)	In triangle ABC , $a = 27$ cm. , $m(\angle B) = 82^\circ$, $m(\angle C) = 56^\circ$, then its surface area $\approx \dots\dots\dots \text{cm}^2$. (a) 540 (b) 447 (c) 350 (d) 400	
13)	In triangle ABC , $m(\angle A) : m(\angle B) : m(\angle C) = 2 : 3 : 4$, $AB = 12$ cm. , then the length of $\overline{AC} = \dots\dots\dots$ cm. (a) 10 (b) 11 (c) 16 (d) 18	
14)	If r is the length of the radius of the circumcircle of the triangle XYZ , then $\frac{y}{2 \sin Y} = \dots\dots\dots$ (a) r (b) $2r$ (c) $\frac{1}{2}r$ (d) $4r$	

15)	In ΔABC , $\sin A = 2 \sin C$, $BC = 6$ cm. , then $AB = \dots\dots\dots$ cm. (a) 2 (b) 3 (c) 4 (d) 6
16)	If the radius length of circumcircle of ΔABC equals 3 cm. and $\sin A + \sin B + \sin C = 2$, then the perimeter of triangle $ABC = \dots\dots\dots$ cm. (a) 6 (b) 9 (c) 12 (d) 24
17)	In any triangle ABC , $\frac{\sin (A + B)}{\sin A + \sin B} = \dots\dots\dots$ (a) 1 (b) $\frac{c}{a + b}$ (c) $\frac{a}{b + c}$ (d) $\frac{b}{a + c}$
18)	In ΔABC , $\frac{a}{a + b} = \frac{\sin A}{\dots\dots\dots}$ (a) $\sin B$ (b) $\sin C$ (c) $\sin A + \sin B$ (d) $\sin A + \sin C$
19)	 In ΔXYZ , if $3 \sin X = 4 \sin Y = 2 \sin Z$, then $X : y : z = \dots\dots\dots$ (a) $2 : 3 : 4$ (b) $6 : 4 : 3$ (c) $3 : 4 : 6$ (d) $4 : 3 : 6$
20)	 ABC is a triangle in which $\frac{\sin A}{3} = \frac{2 \sin B}{5} = \frac{\sin C}{4}$, then $a : b : c = \dots\dots\dots$ (a) $6 : 5 : 8$ (b) $8 : 5 : 6$ (c) $7 : 2 : 4$ (d) $3 : 5 : 4$
21)	In ΔABC , $a - b = 4$ cm. , $\sin A = \frac{3}{2} \sin B$, then $a = \dots\dots\dots$ cm. (a) 4 (b) 6 (c) 8 (d) 12
22)	 In ΔABC , $\frac{2b}{\sin B} = \dots\dots\dots r$ (where r is the radius of its circumcircle) (a) 1 (b) 2 (c) 4 (d) 8

كيفية طباعة صفحات معينة من ملف معين مثلا ازاي نطبع الصفحات من صفحة 4 الى صفحة 9

